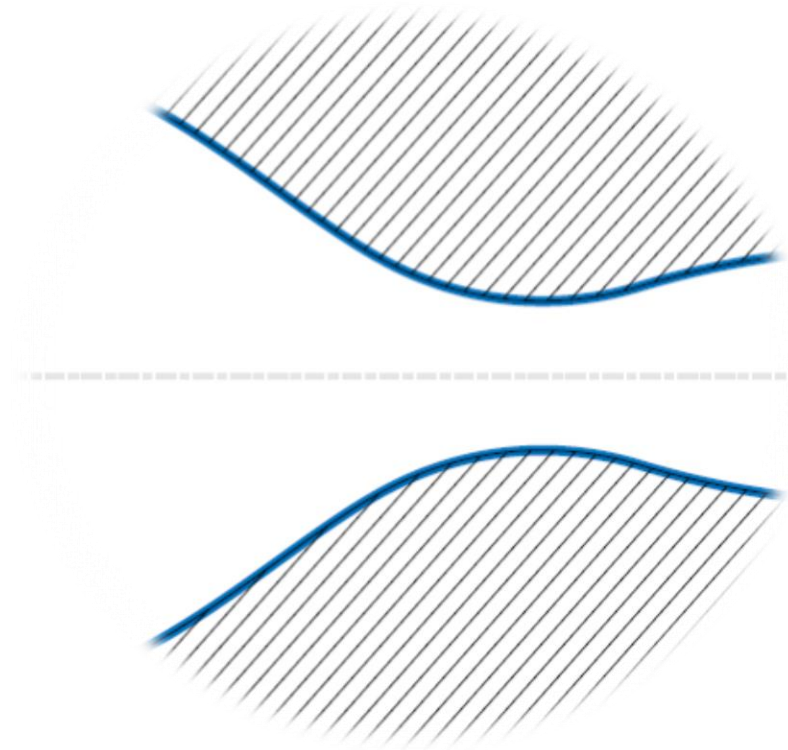


# *Similarity Parameters for Non-Ideal One-Dimensional Isentropic Expansions*

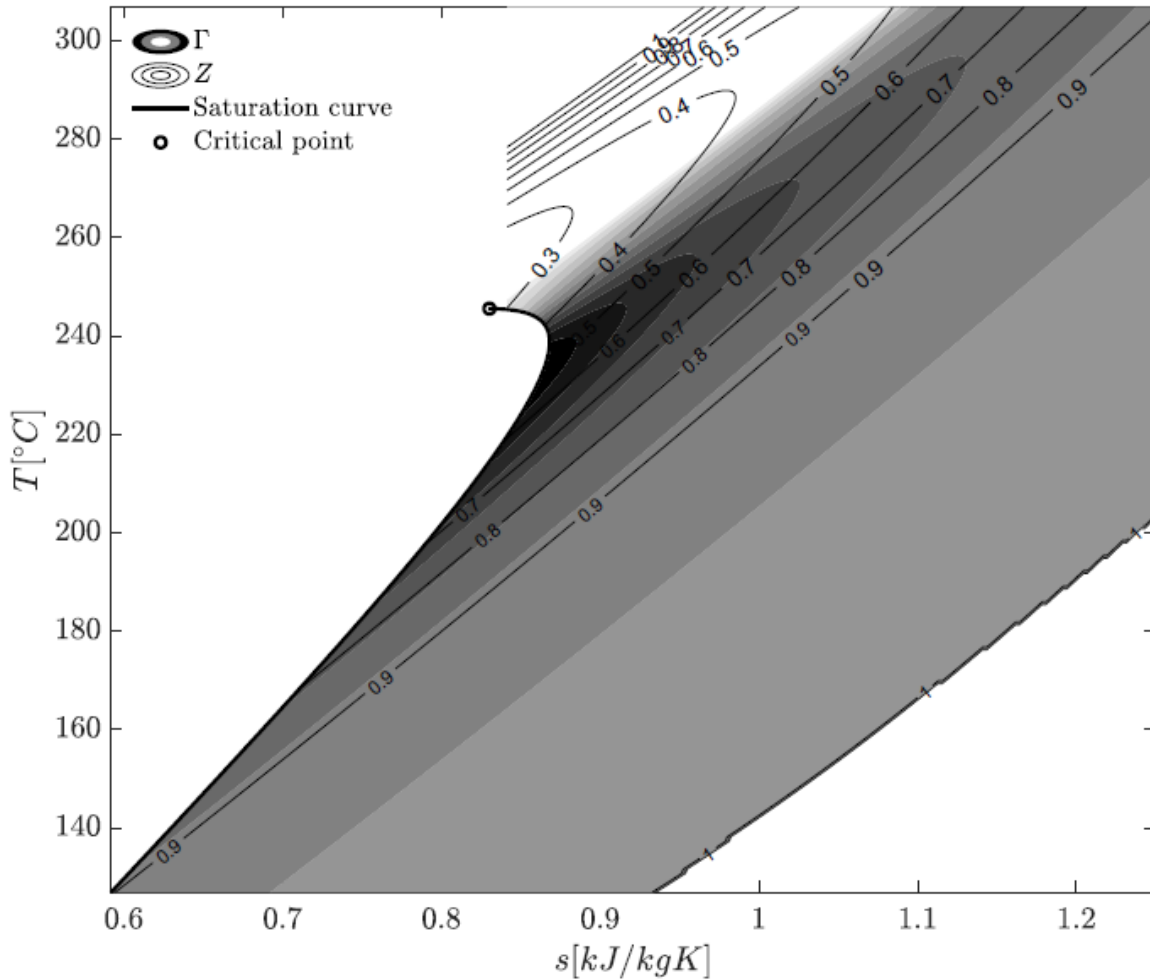
**C. C. Conti\***, A. Spinelli, A. Guardone

\*camillacecilia.conti@polimi.it



## Non-Ideal

# Compressible Fluid Dynamics (**NICFD**)

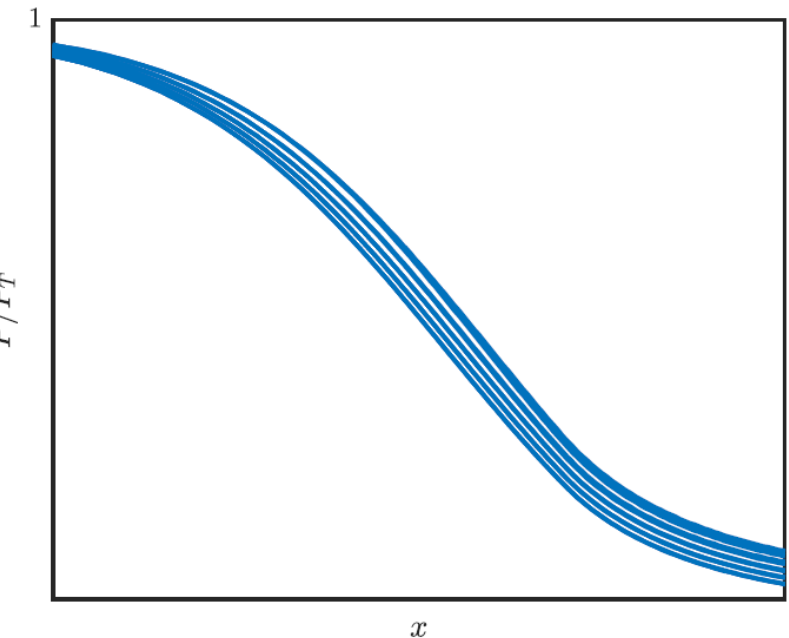
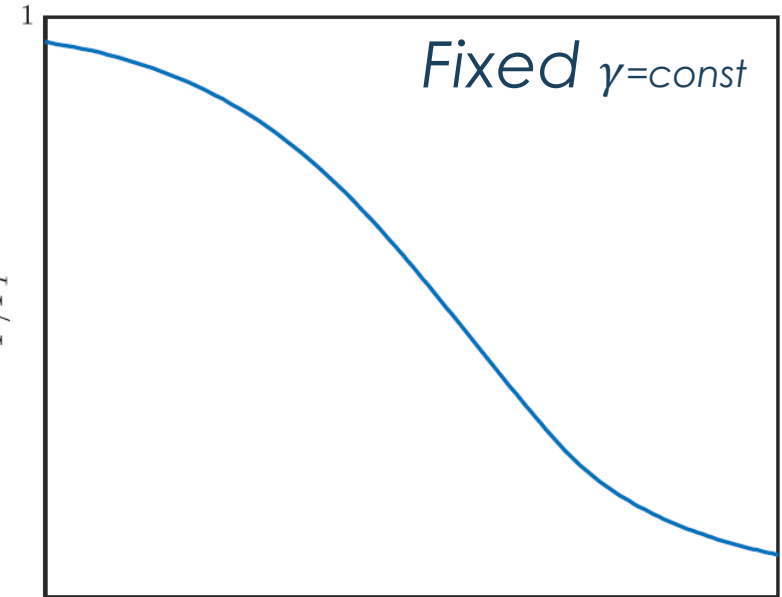
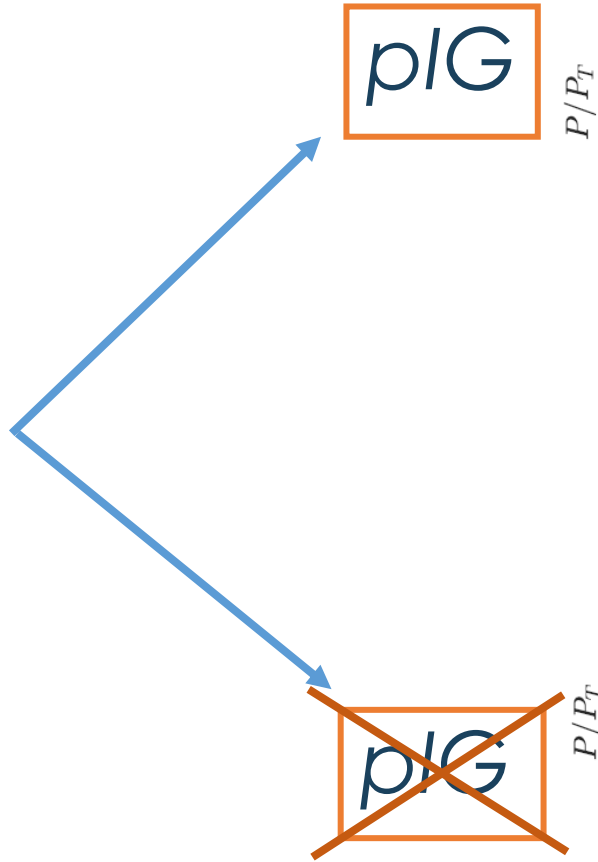
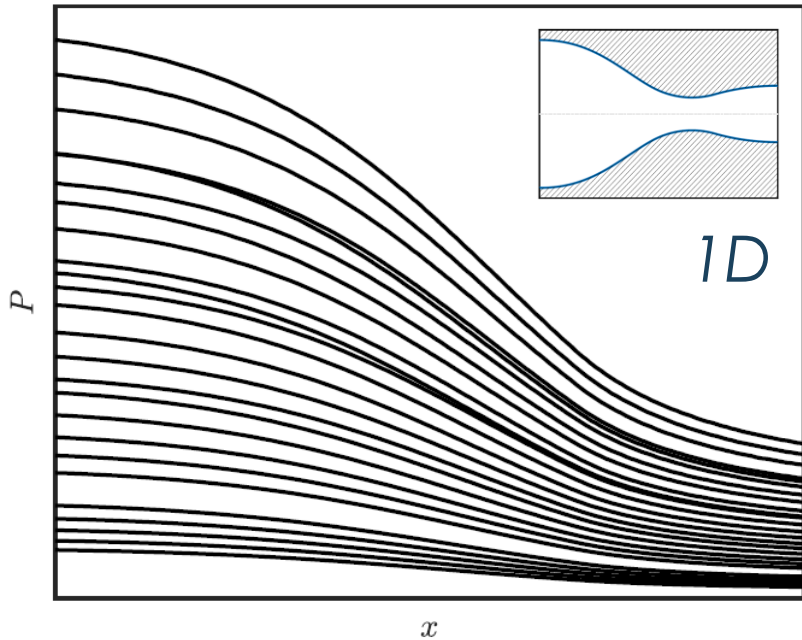


Dense vapors  
Close to VLE + critical point  
Two-phase

$$Pv \neq RT$$

# Focus on: Non-Ideal Dependence of Isentropic Expansions

$$\frac{P}{P_T} = f(P_T, T_T)$$

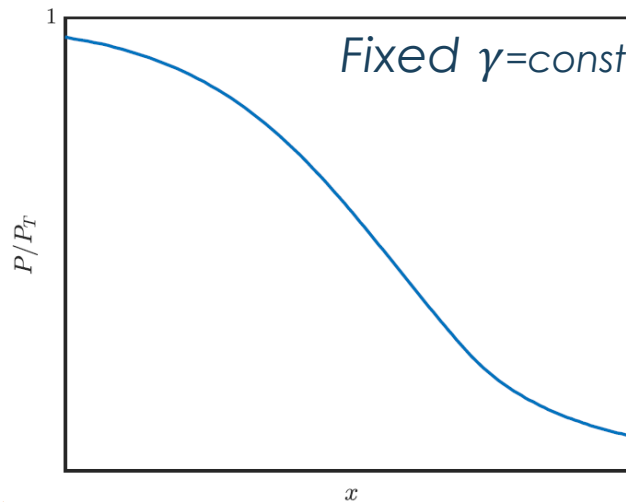


# Thermal and Caloric Behaviour

Thermodynamic description (pure fluid)

Thermal EOS  
 $P = P(T, v)$

Caloric EOS  
 $e = e(T, v)$



$$Z = \frac{P}{R T \rho}$$

vs. IG at same  $(T, P)$

IG

$$Z = 1$$

Same volumetric behaviour  
 regardless of  $(P_T, T_T)$

$$\Gamma = 1 + \frac{\rho}{c} \left( \frac{\partial c}{\partial \rho} \right)_s$$

$c$  in *iso* - *s* process +  
 molecular complexity

$$\Gamma = \frac{\gamma + 1}{2}$$

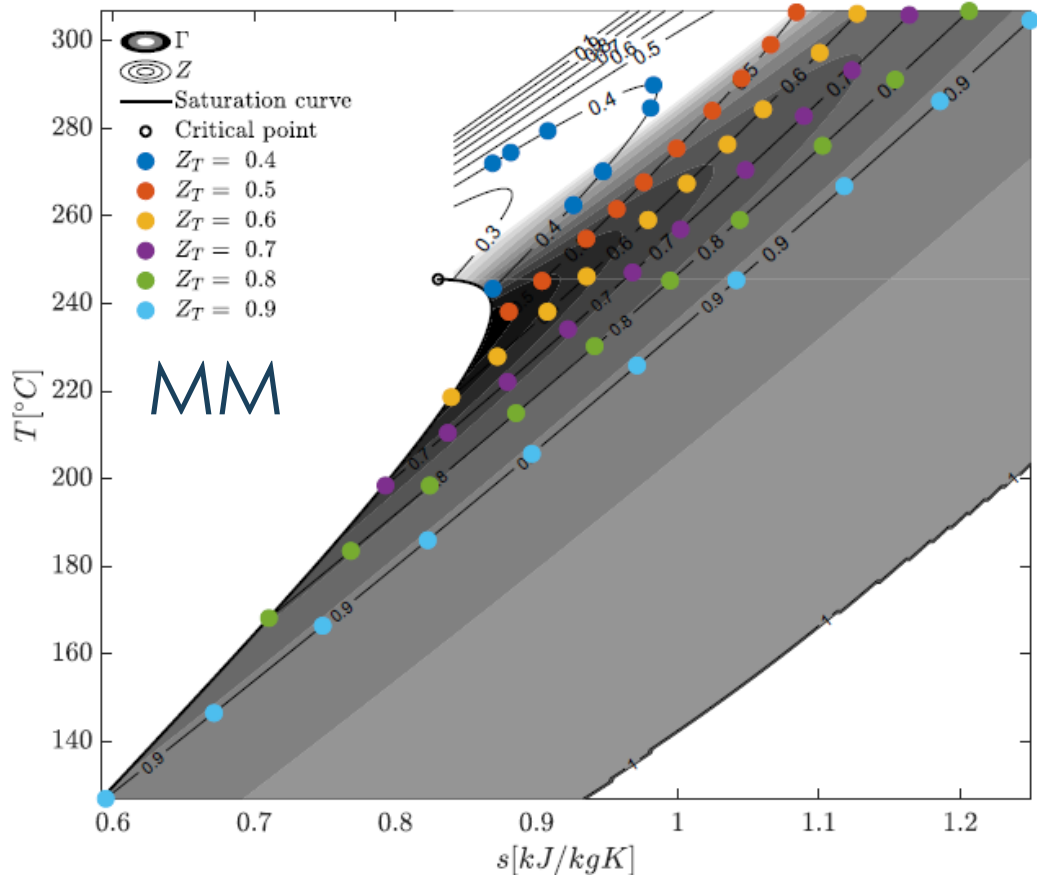
pIG

Same caloric behaviour  
 regardless of  $(P_T, T_T)$   
 for fixed fluid



***Z,  $\Gamma$  in Non-ideal flows?***

# Calculation Framework: 1D isentropic expansions

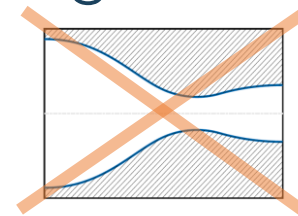


Type	Fluid
Siloxanes	MM
	MDM
	D4
	D6
Alkanes	Butane
	Pentane
	Hexane
	Octane
	Decane
Cyclic Alkanes	Cyclopentane
	Cyclohexane
Alkenes	Isobutene
	Benzene
Other Hydrocarbons	Toluene
	Acetone
Halocarbons	R1234yf
	R218
	R1233zd
	R227ea
	RC318
Other Fluids	Water
	Carbon Dioxide Ammonia

- $10 \times (P_T, T_T)$  with same  $Z_T = \frac{P_T}{R T_T \rho_T}$
- $0.4 \leq Z_T \leq 0.9$
- widest possible region
- $P_{max} = 3P_C, T_{max}$  from TDM

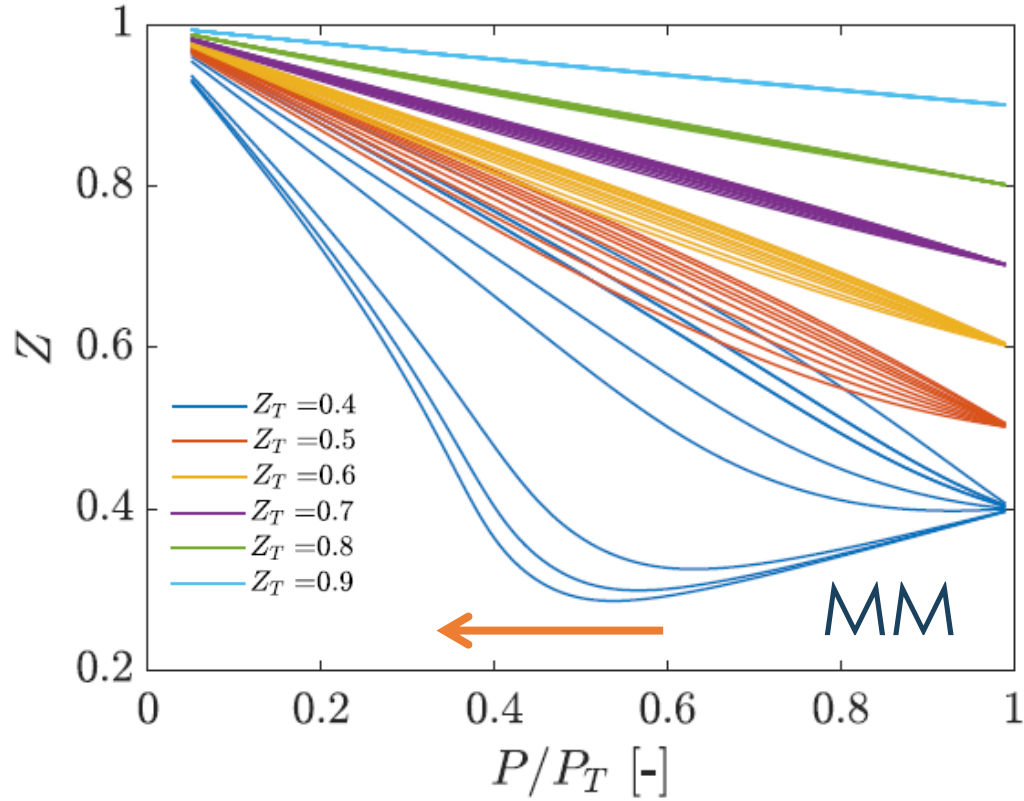
- Helmholtz energy-based TDM
- Independent of geometry:

$$\frac{P}{P_T} : 1 \rightarrow 0.05$$

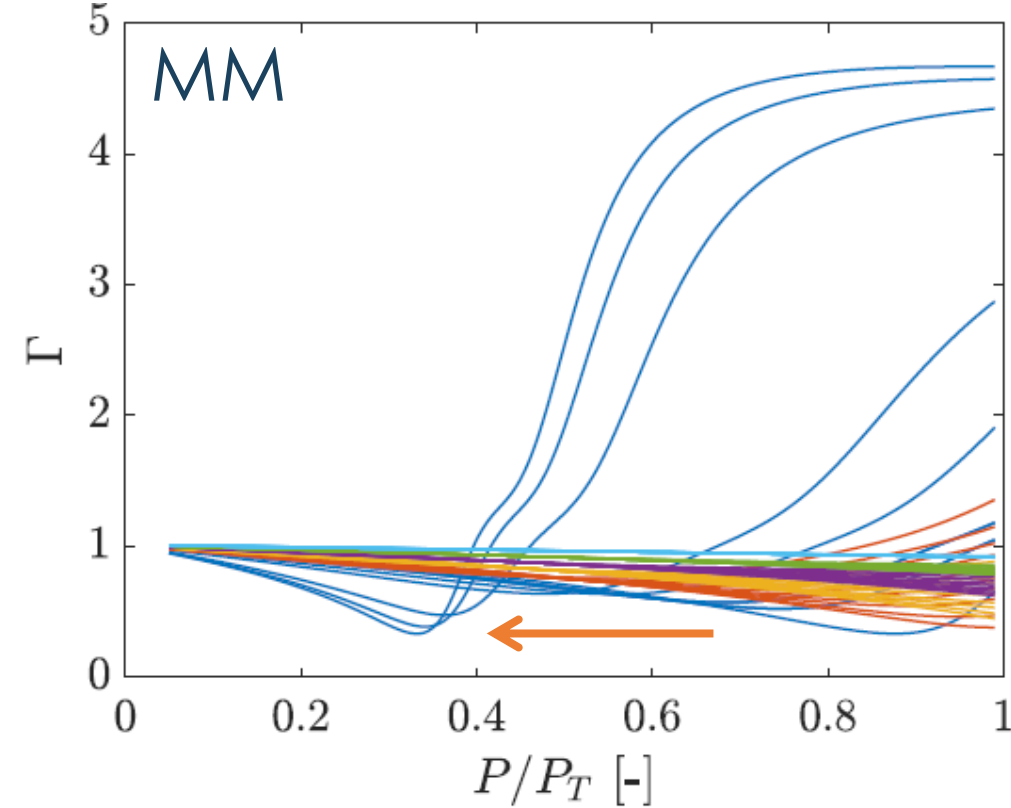


- 20 HMC fluids
- 3 LMC fluids

# Results: $Z$ and $\Gamma$ Similarity



$$Z_T = \frac{P_T}{R T_T \rho_T}$$



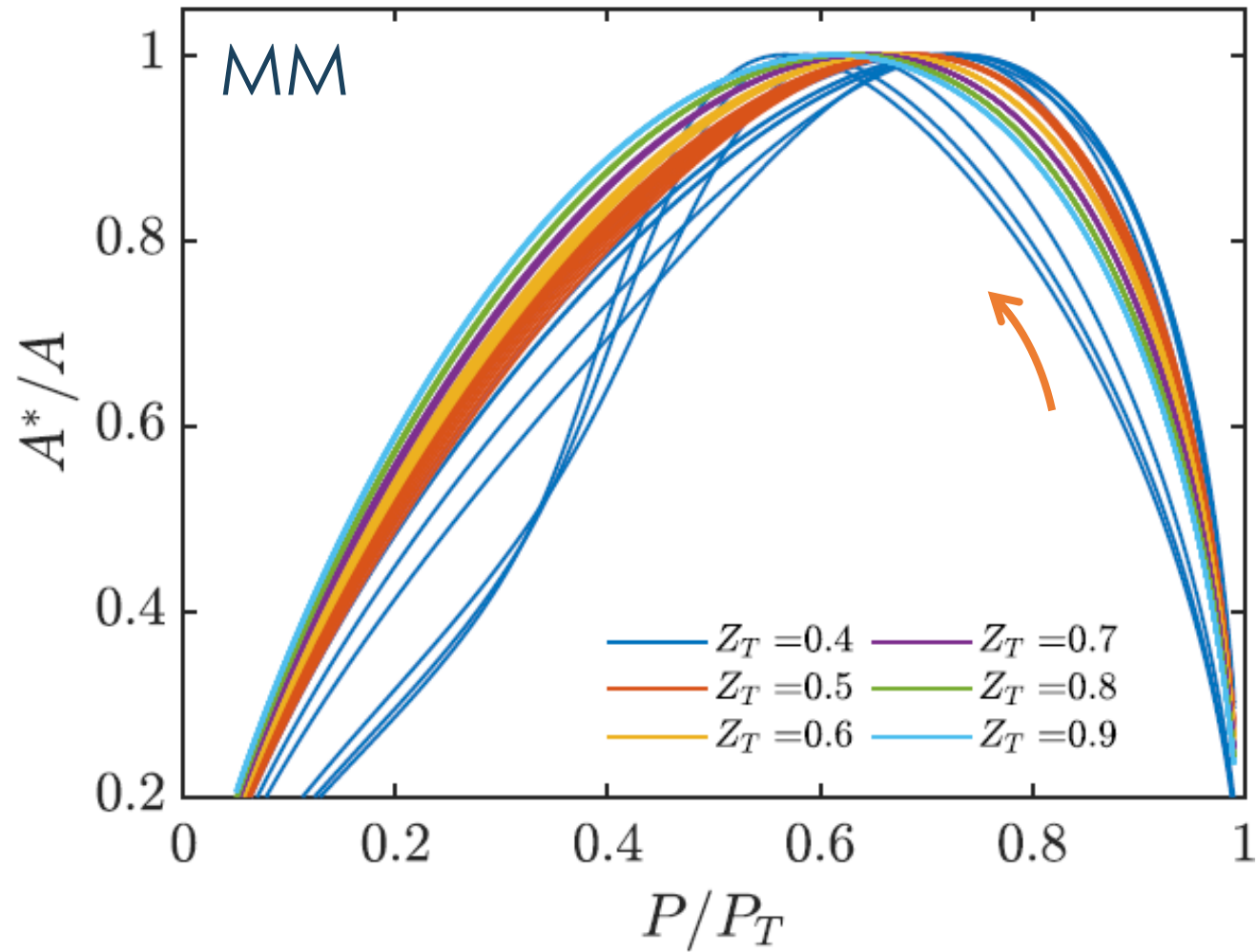
- MM (HMC)
- Same  $Z_T \rightarrow$  similar  $Z$  and  $\Gamma$  along expansions



Similar **volumetric** + **caloric** behavior!

- $\downarrow Z_T \rightarrow$  less similar  $Z$  and/or  $\Gamma$

# Results: Expansions Similarity

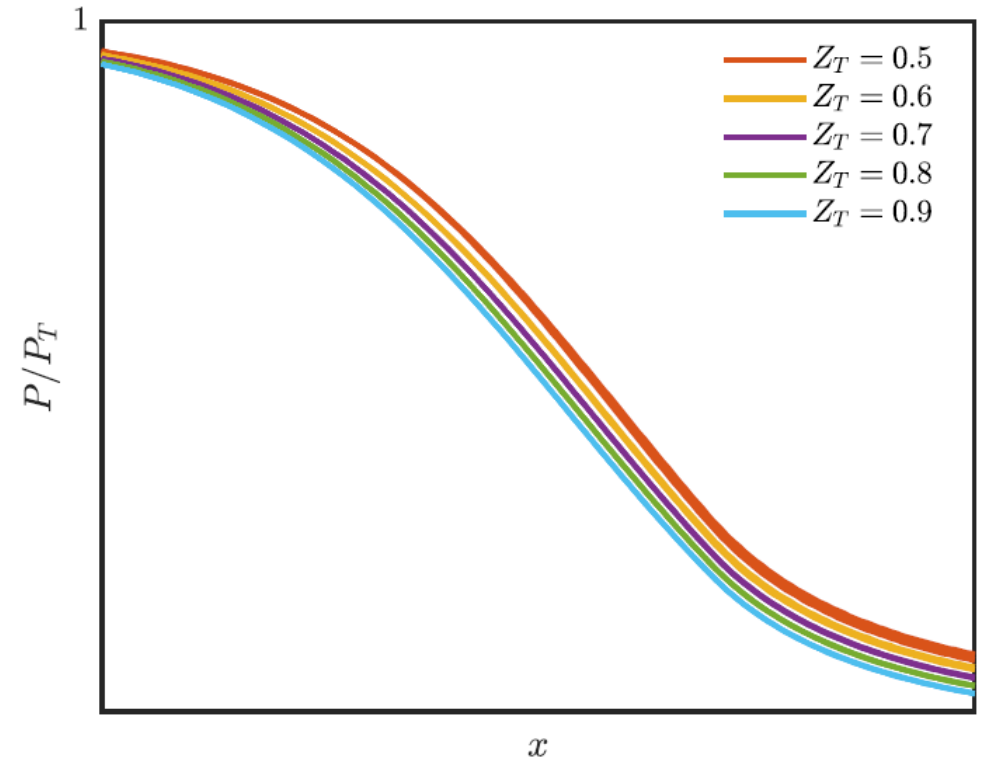
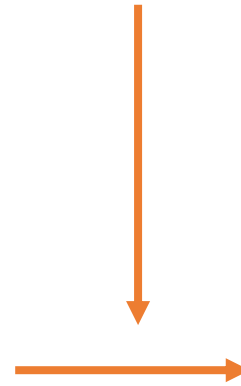
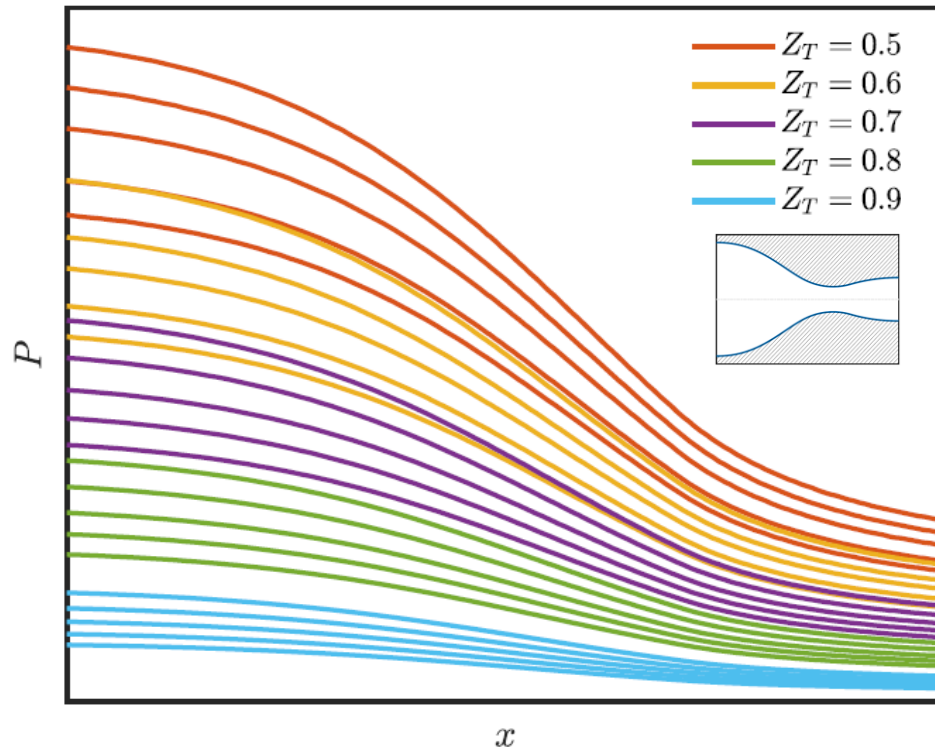


- MM (HMC)
- Expansions grouped according to  $Z_T$  (except  $Z_T = 0.4$ )

# Results: $Z_T$

## Similarity Parameter to Characterize Non-Ideal Isentropic Expansions (HMC)

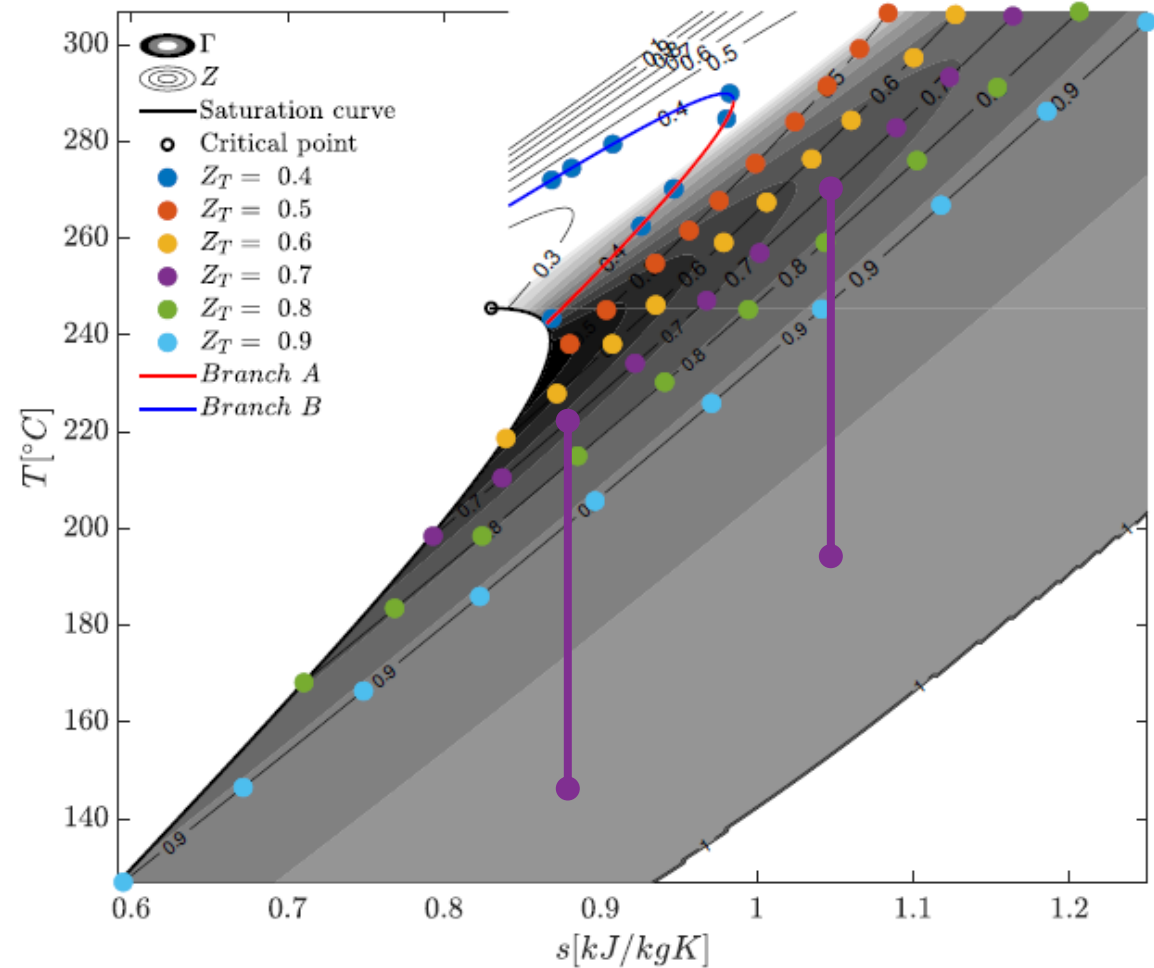
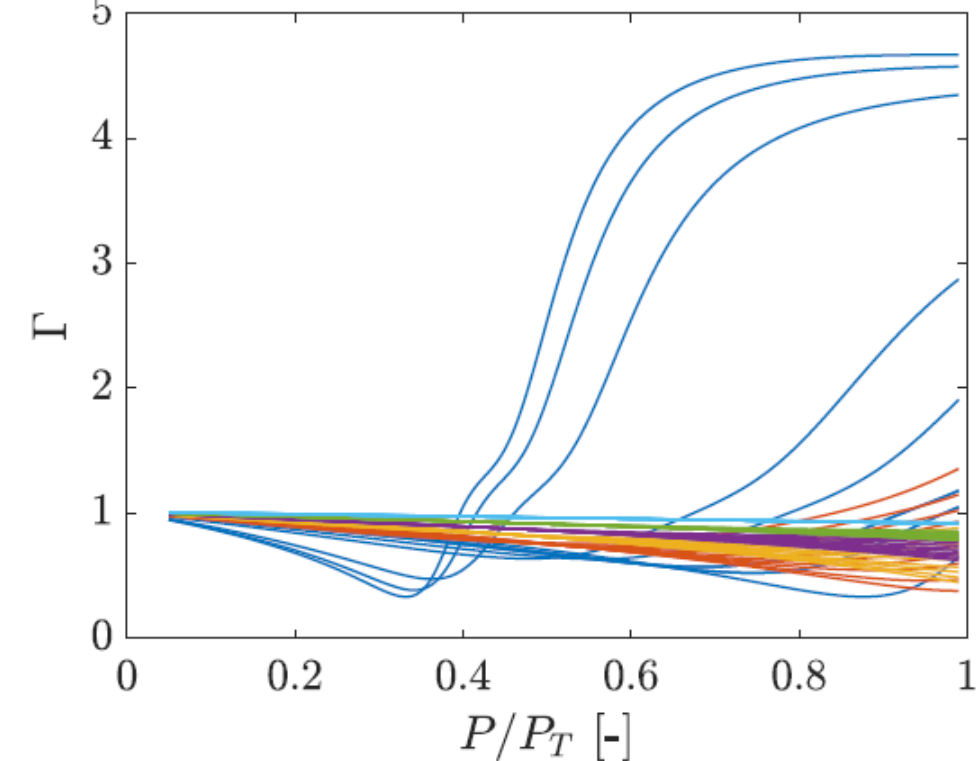
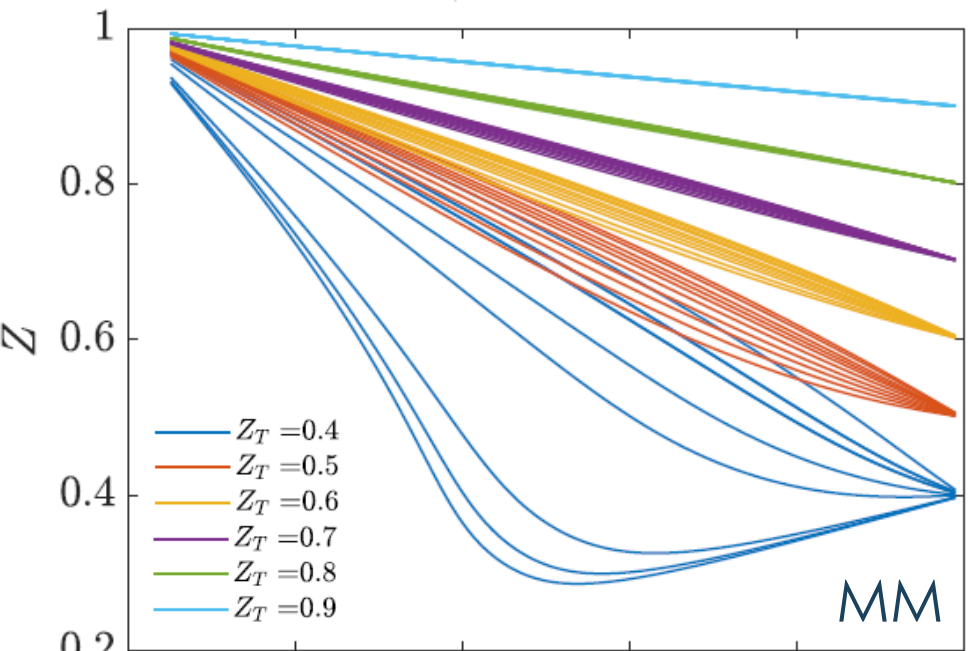
$$Z_T = \frac{P_T}{R T_T \rho_T}$$



- Experimentally verified in the TROVA!



# Results: $Z$ and $\Gamma$ along expansions (HMC - MM)

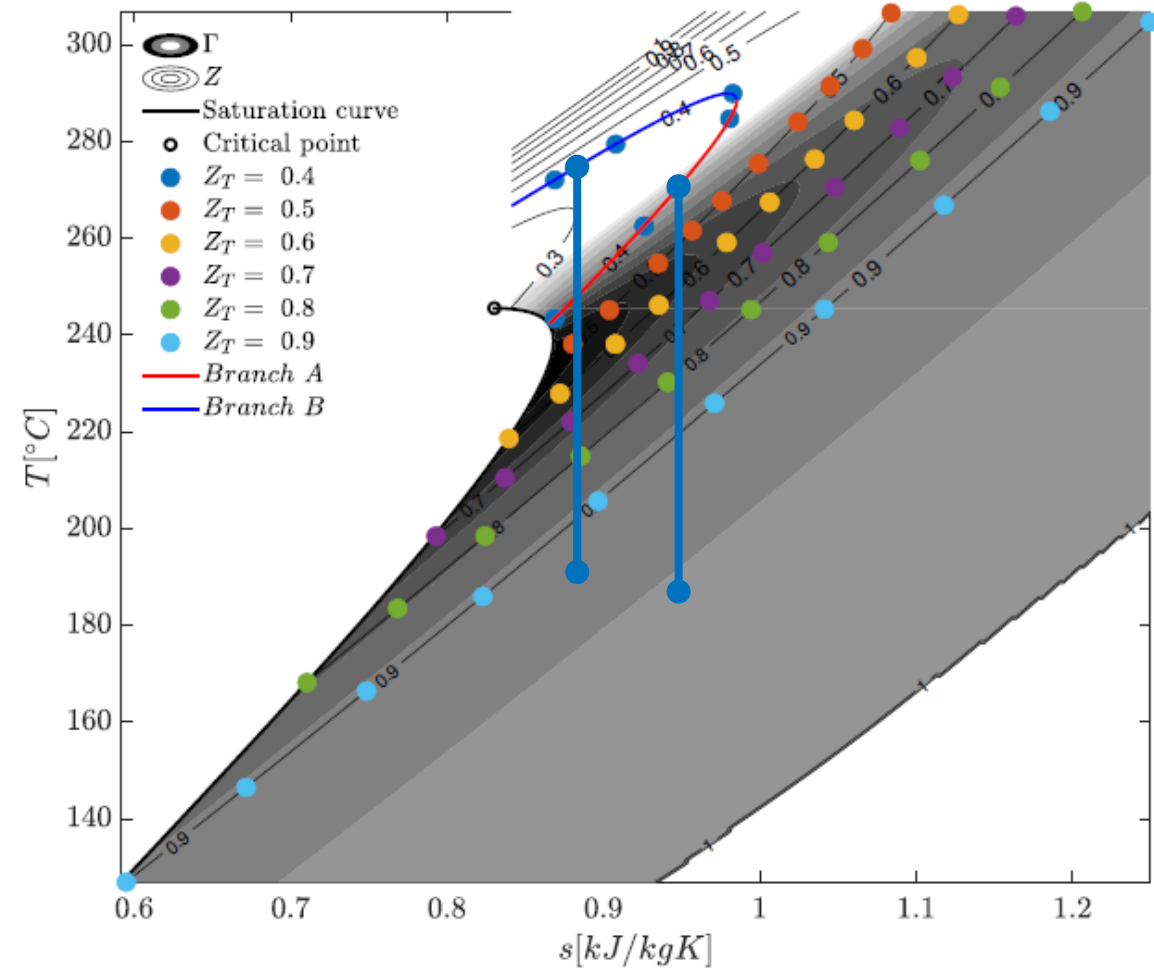
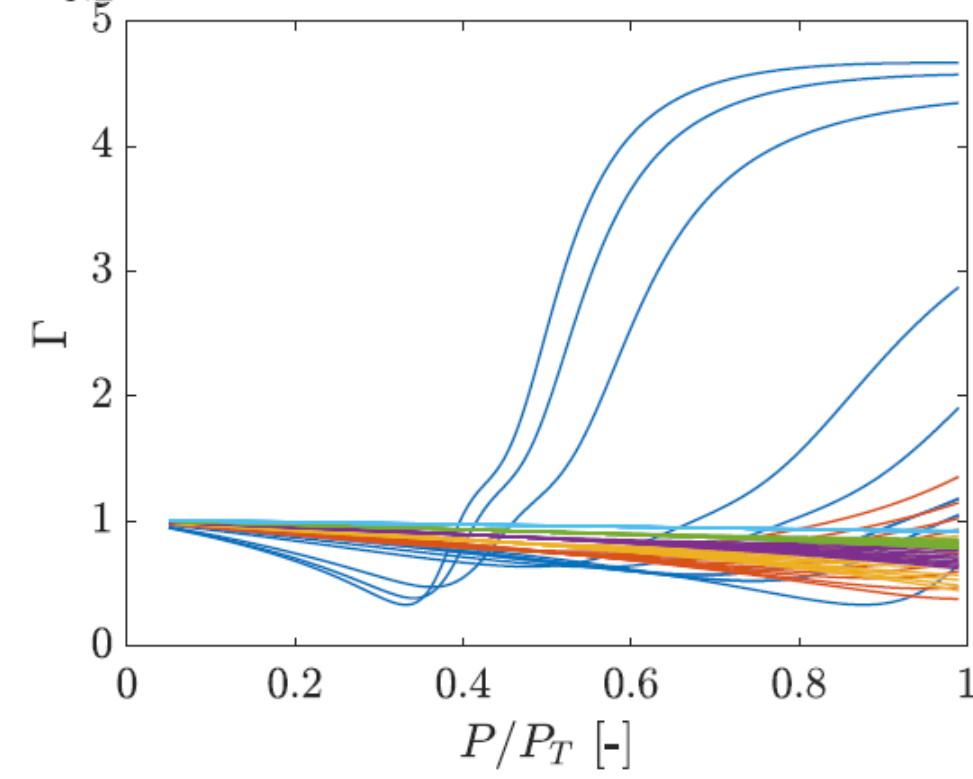
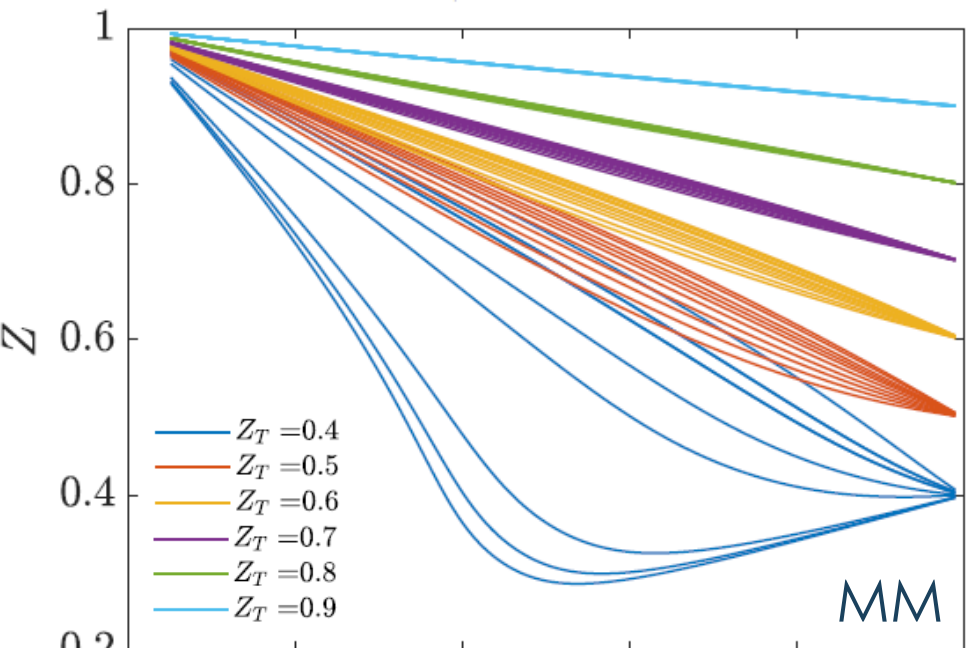


Far from critical point:  
*iso* -  $Z$  // to one another  
*iso* -  $\Gamma$  // to one another  
*iso* -  $Z$  // *iso* -  $\Gamma$



Same  $Z_T$ :  
 similar  $Z$  and  $\Gamma$   
 similar expansions

# Results: $Z$ and $\Gamma$ along expansions (HMC - MM)



Less ideal conditions:  
Branch A  
Branch B

(both *iso-Z* and *iso- $\Gamma$* )

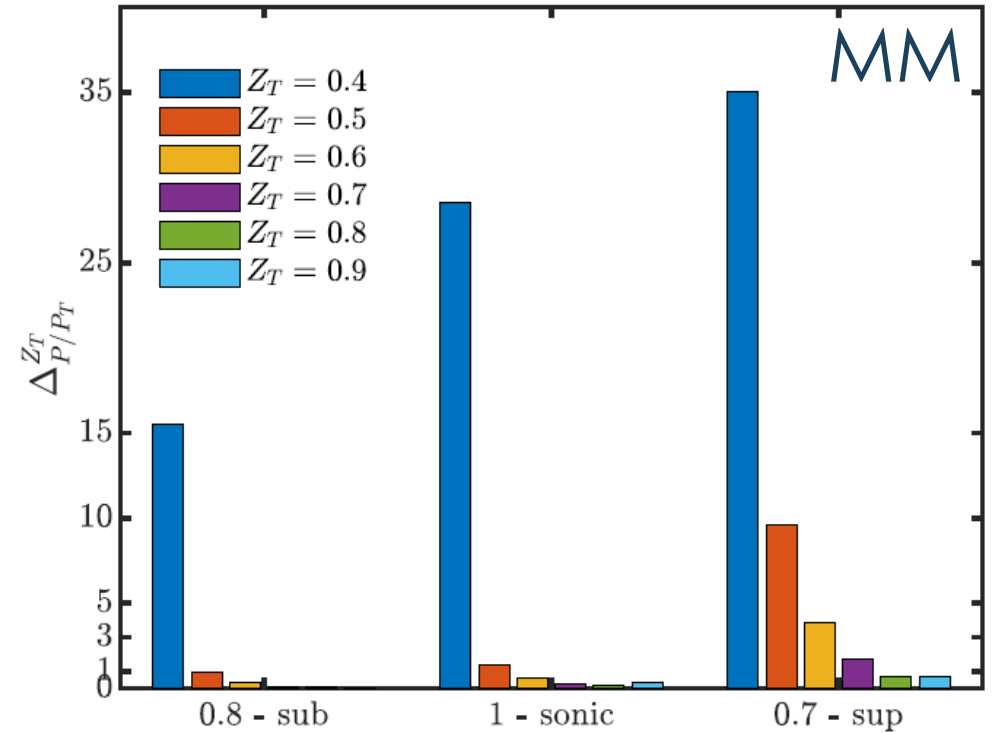
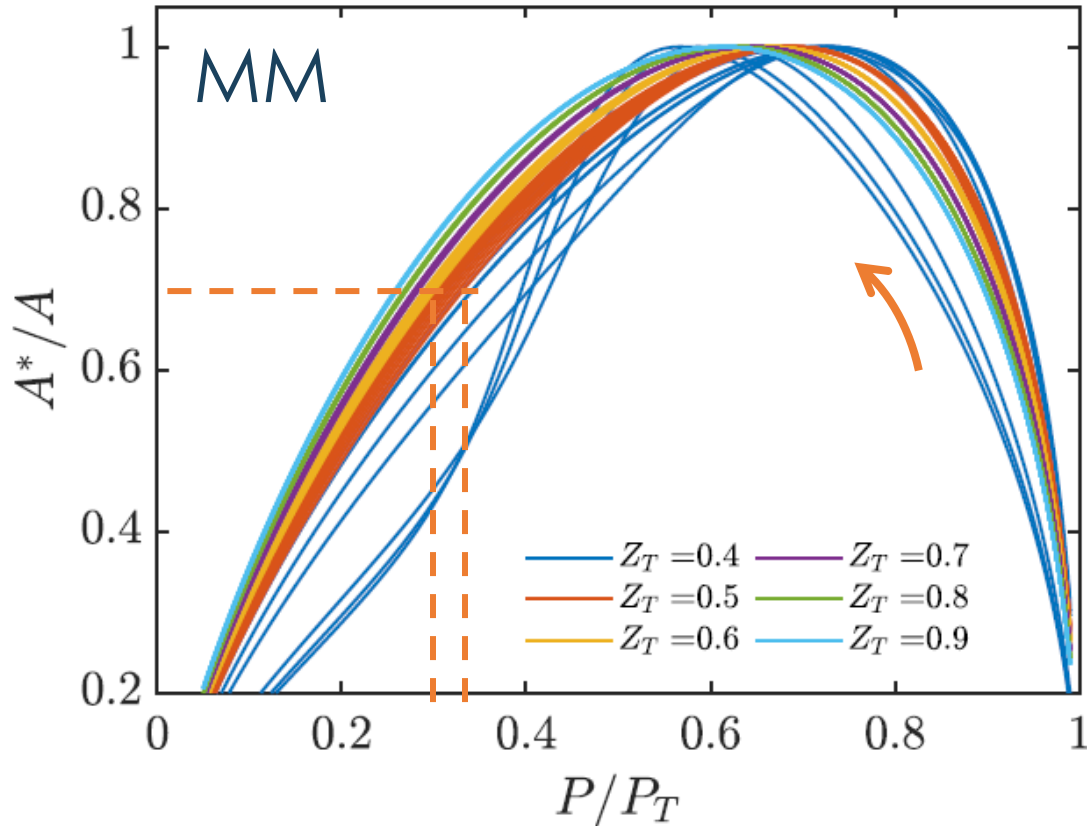


Even at same  $Z_T$ :  
Not similar  $Z$  and  $\Gamma$   
Not similar expansions

# Results: $Z_T$ as a Similarity Parameter for HMC fluids

Spread in  $P/P_T$  in a 'fixed geometry' for expansions with same  $Z_T$  but different  $(P_T, T_T)$

$$\Delta_{P/P_T}^{Z_T} = \frac{\max P/P_T(Z_T) - \min P/P_T(Z_T)}{\min P/P_T(Z_T)} \cdot 100$$



$Z_T$  as similarity parameter ✓  
Well representative of HMC

- $\downarrow \Delta_{P/P_T}^{Z_T}$  if  $\uparrow$  ideality
- $\uparrow \Delta_{P/P_T}^{Z_T}$  if  $M \uparrow$  ( $\downarrow$  denominator)

# Conclusions & Future Outlook

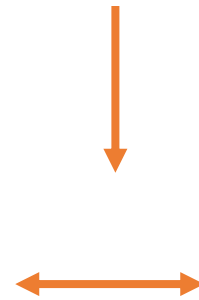
Non-ideal 1D isentropic expansions → **Similarity Parameter?**

- HMC fluids +  $Z_T \geq 0.5$

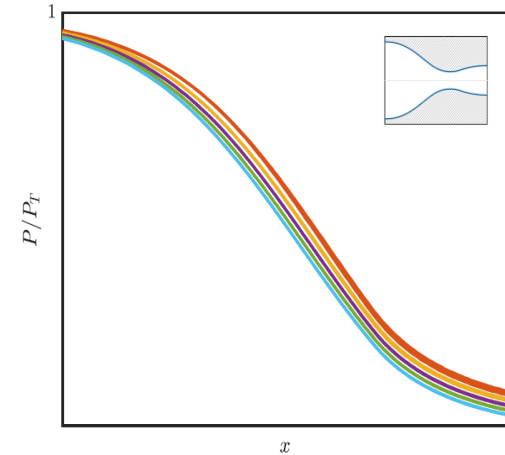
- similar  $Z$  and  $\Gamma$  (*iso* –  $Z$ , *iso* –  $\Gamma$  shape)

- similar  $\frac{P}{P_T}$

Similar thermal  
+  
caloric behaviour



Similar expansions



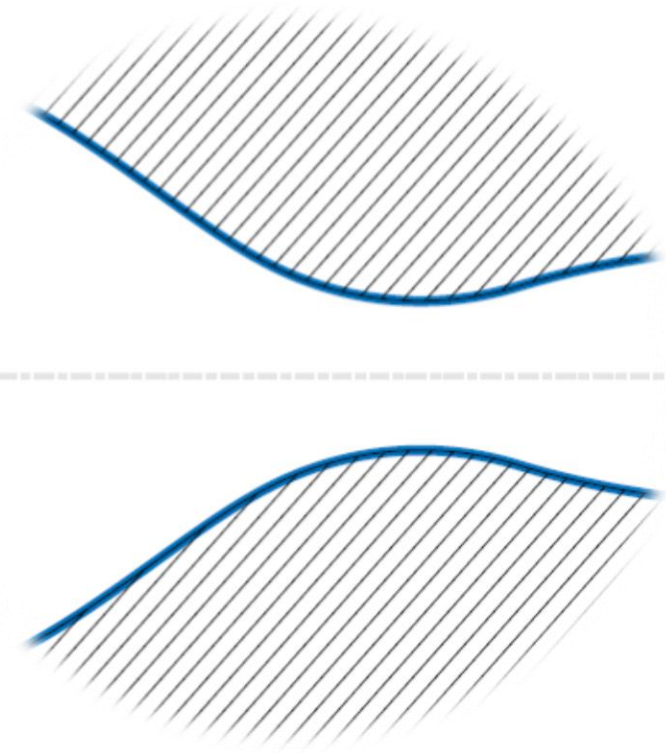
- VdW calculation
- Molecular interpretation



**POLITECNICO**  
MILANO 1863



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Established by the European Commission

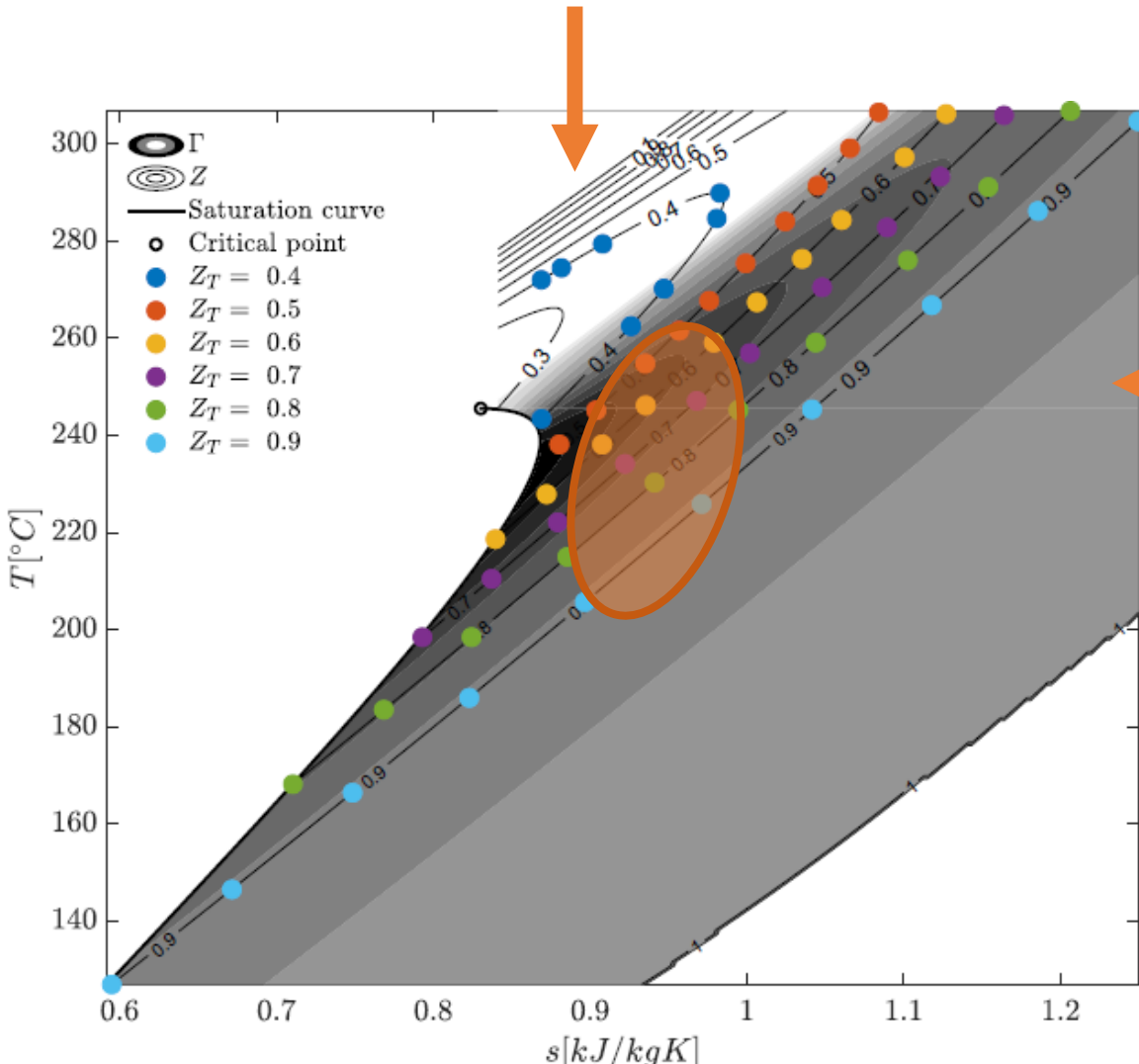


**NICFD 2020**  
for Propulsion & Power

*3rd International Seminar on Non-Ideal Compressible-Fluid Dynamics for Propulsion & Power, 29-30 Oct. 2020 – Delft*

# Objective

- Experimentally observed (TROVA): expansions within errorbars for  $Z_T > 0.6$   
Siloxane MM + limited region

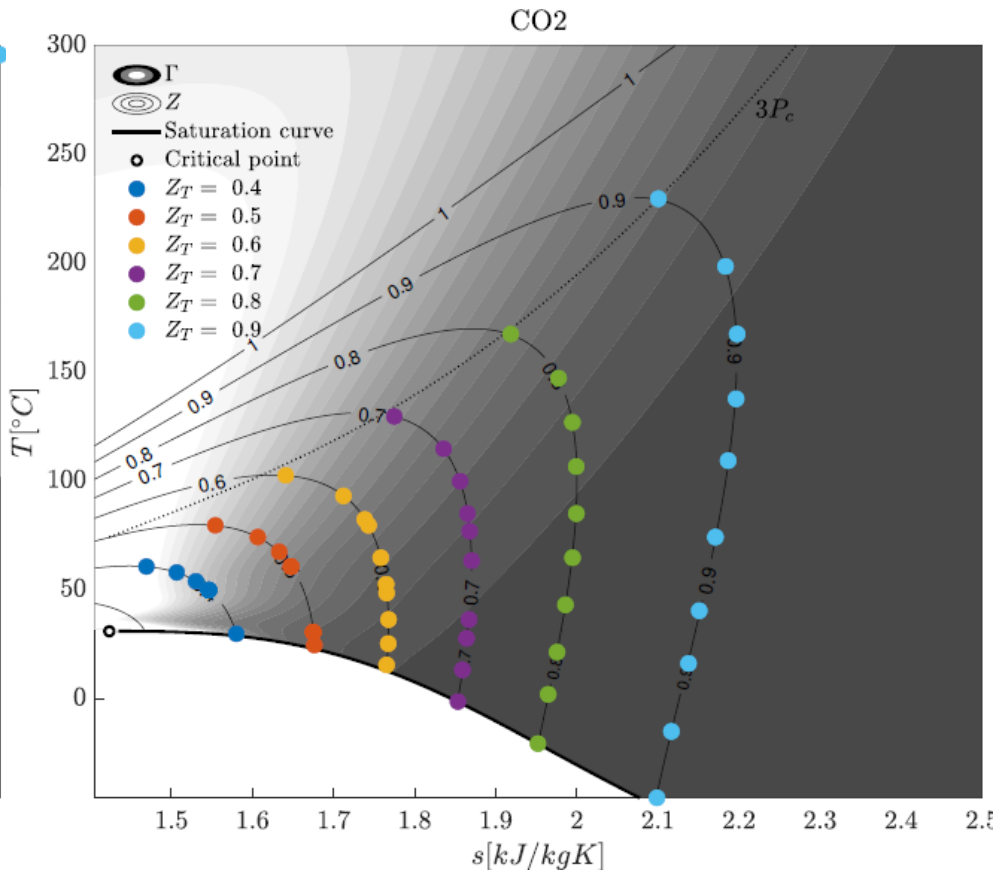
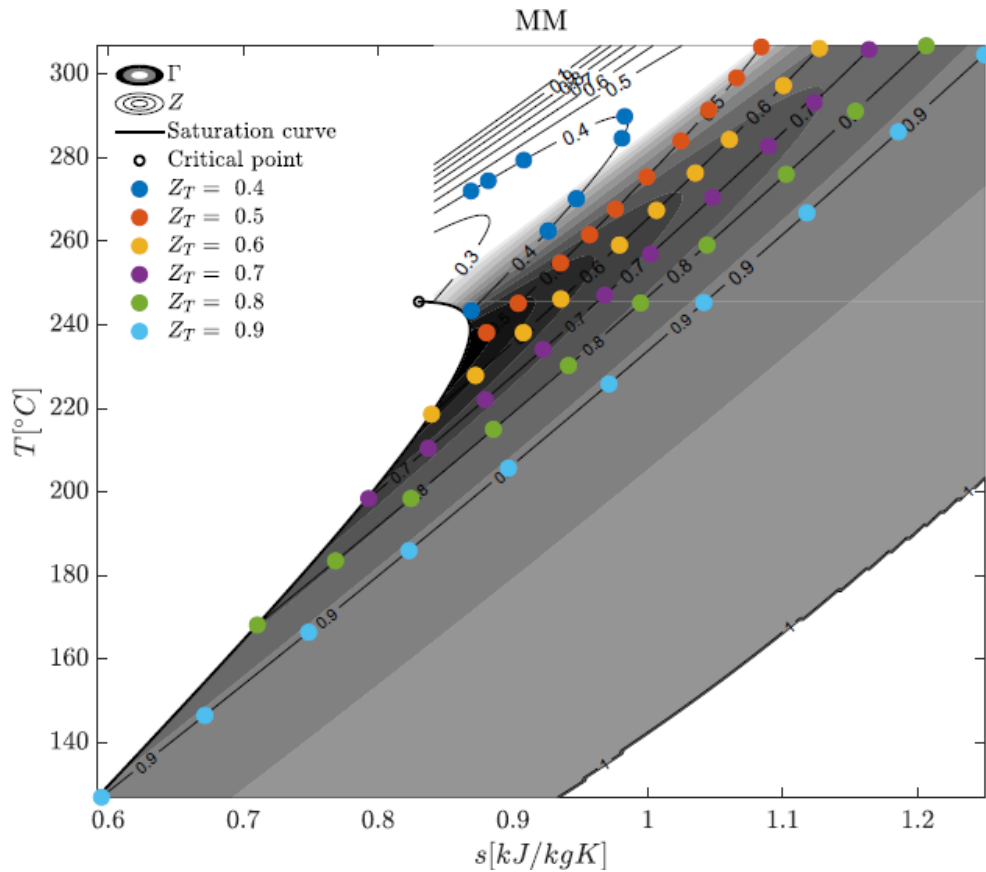


Suitability of  $Z_T$  as similarity parameter

- Wider region
- Other fluids

**1D Isentropic Expansions**

# Calculation Framework: 1D isentropic expansions

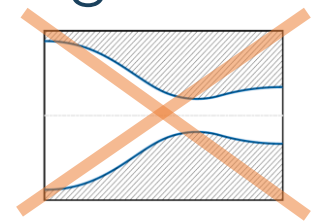


Type	Fluid
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	MDM
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	D6
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	Acetone
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	R1233zd
	R227ea
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Other Fluids	Water
	Carbon Dioxide Ammonia

- $10 \times (P_T, T_T)$  with same  $Z_T = \frac{P_T}{R T_T \rho_T}$
- $0.4 \leq Z_T \leq 0.9$
- widest possible region
- $P_{max} = 3P_c, T_{max}$  from TDM

- Helmholtz energy-based TDM
- Independent of geometry:

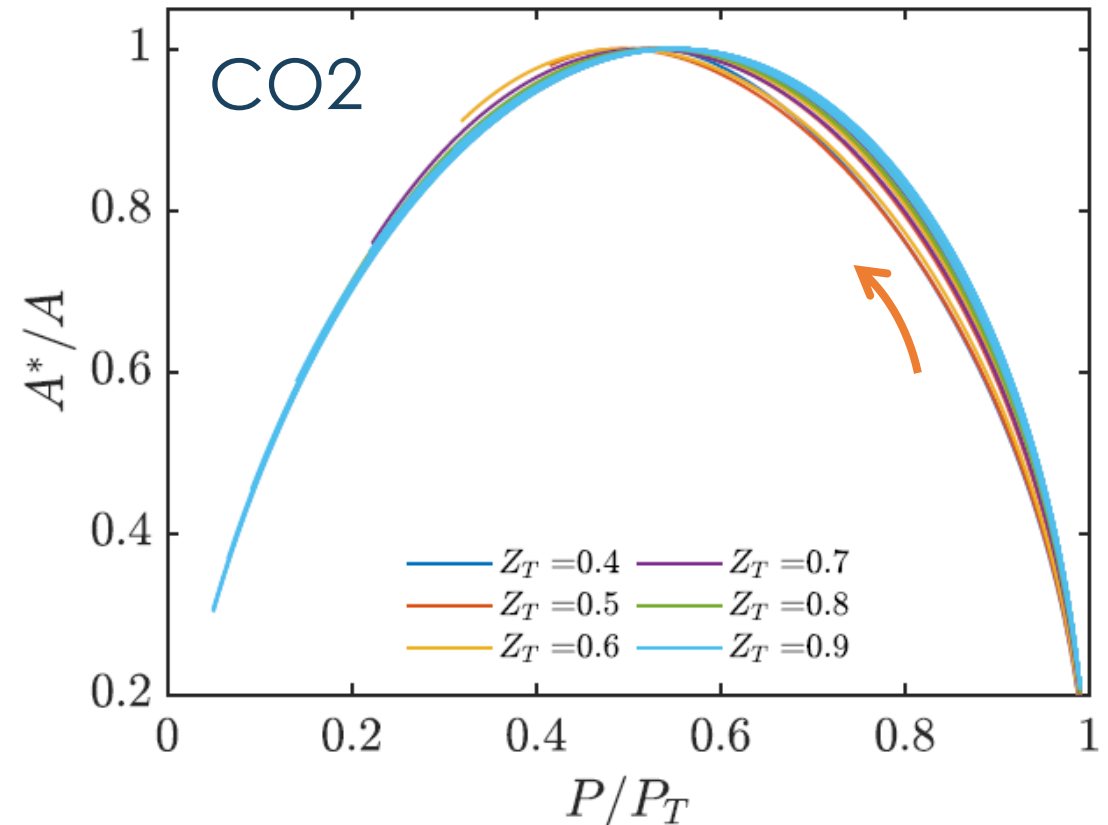
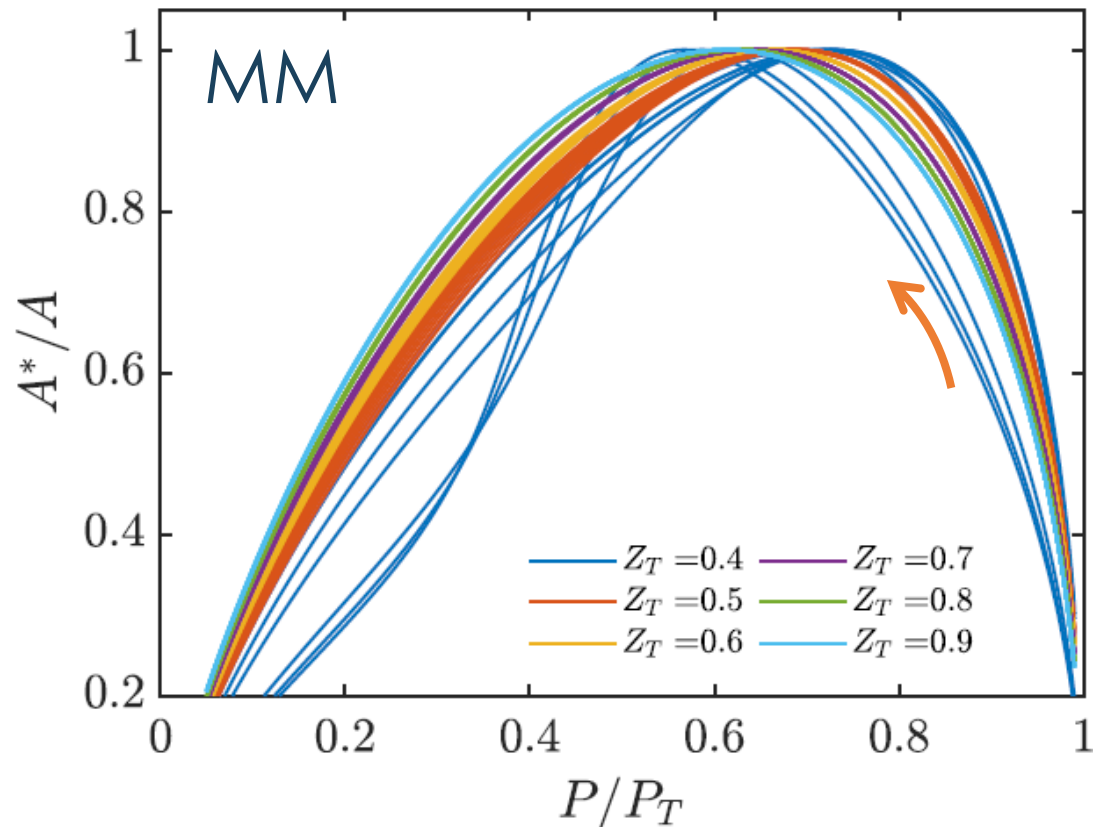
$$\frac{P}{P_T} : 1 \rightarrow 0.05$$



- 20 HMC fluids
- 3 LMC fluids

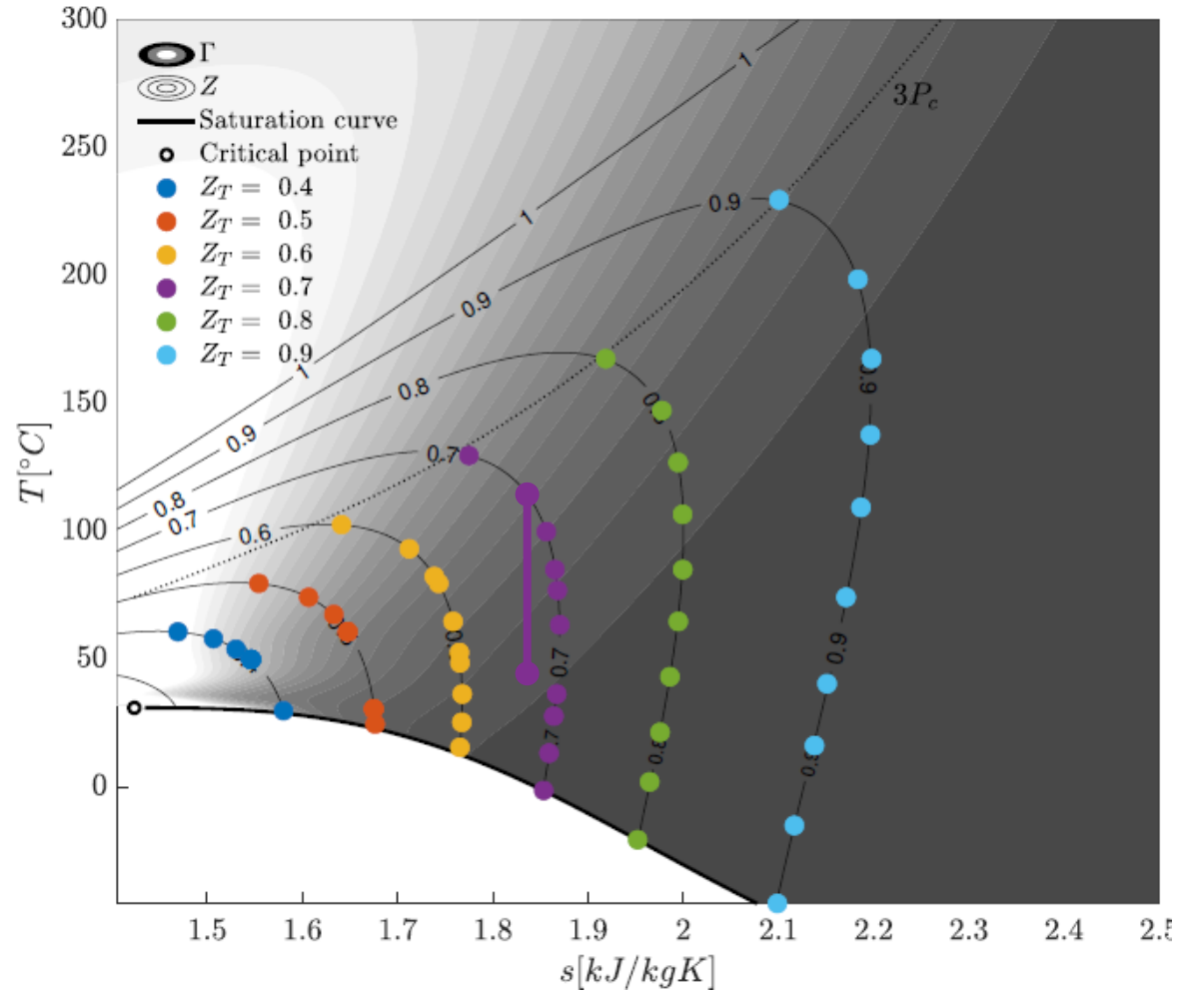
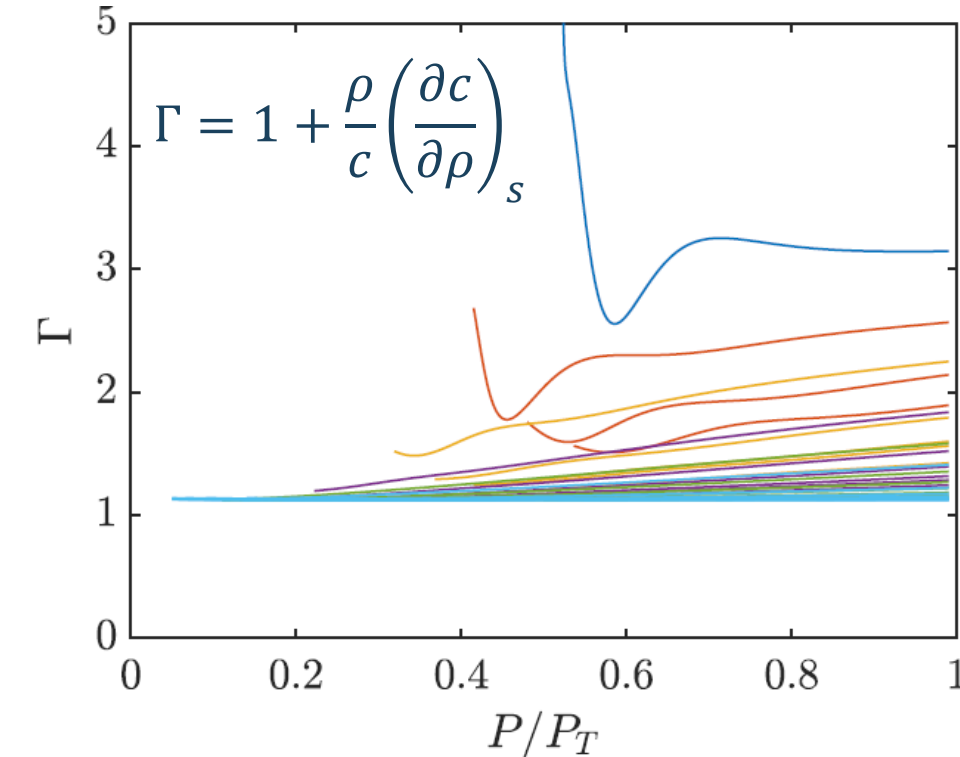
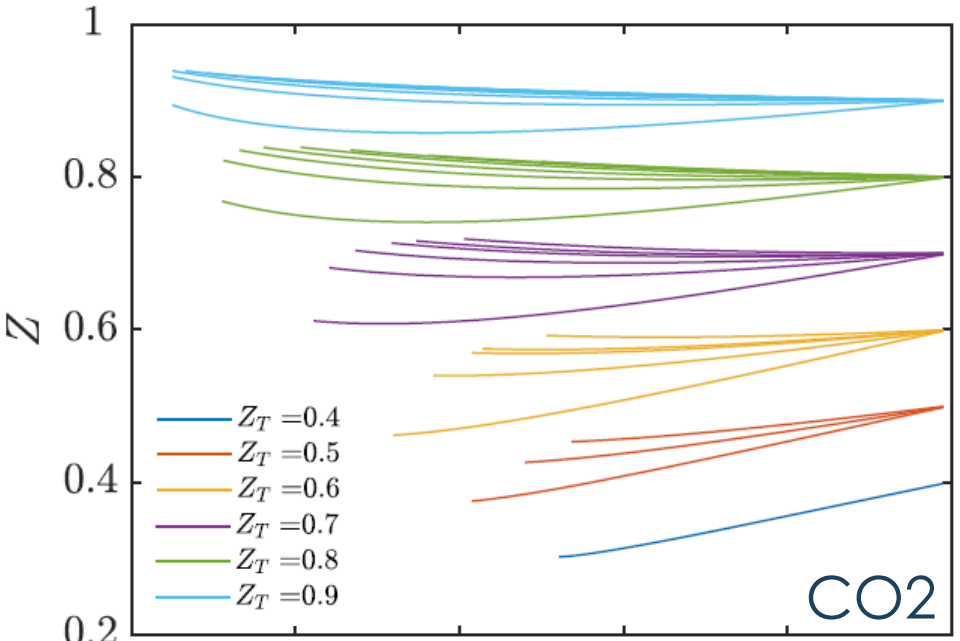
# Results: $P/P_T$ Similarity

- MM (HMC) and CO2 (LMC)
- Confirmed  $\frac{P}{P_T} = f(P_T, T_T)$
- Expansions grouped according to  $Z_T$  for HMC (except  $Z_T = 0.4$  )



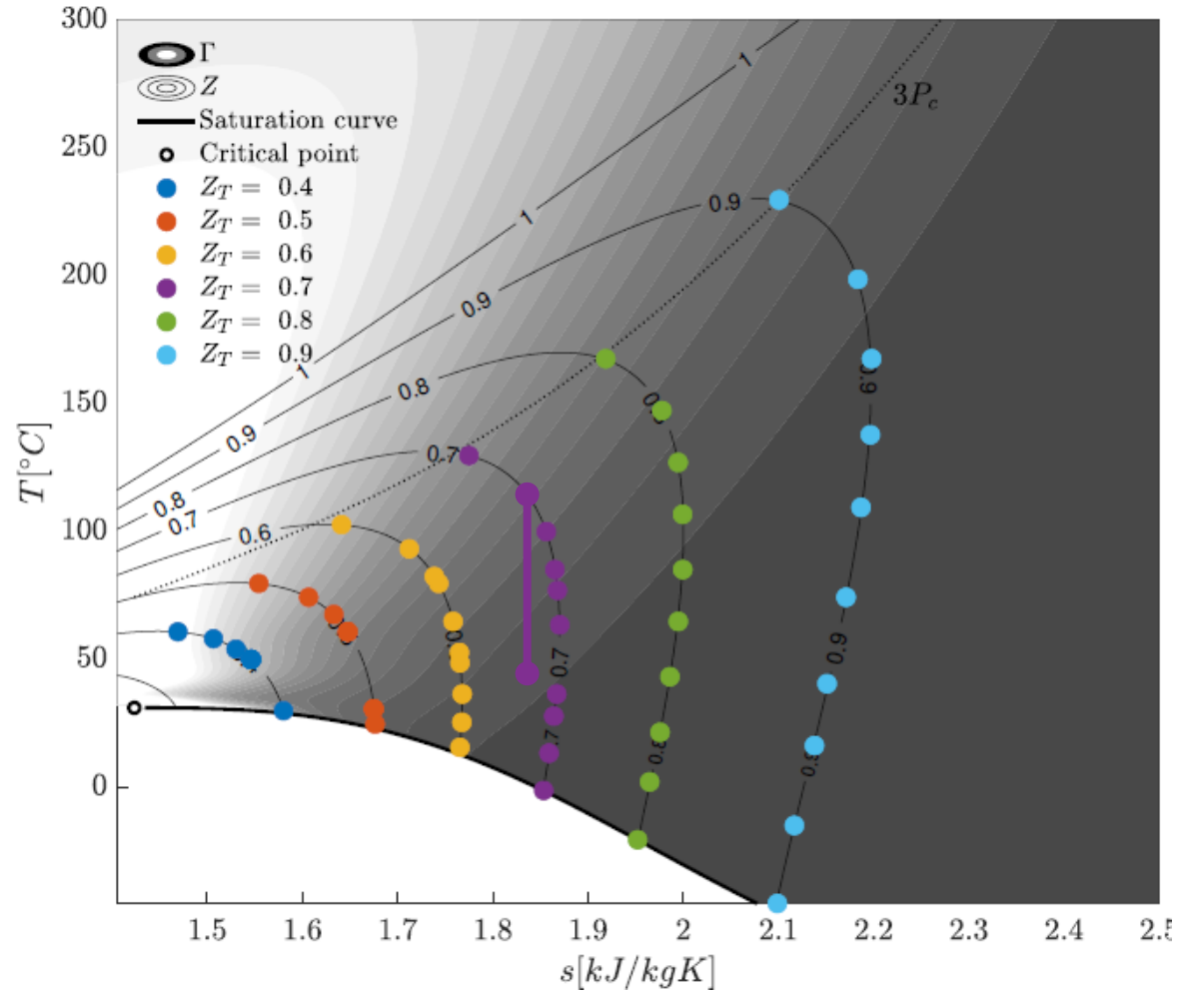
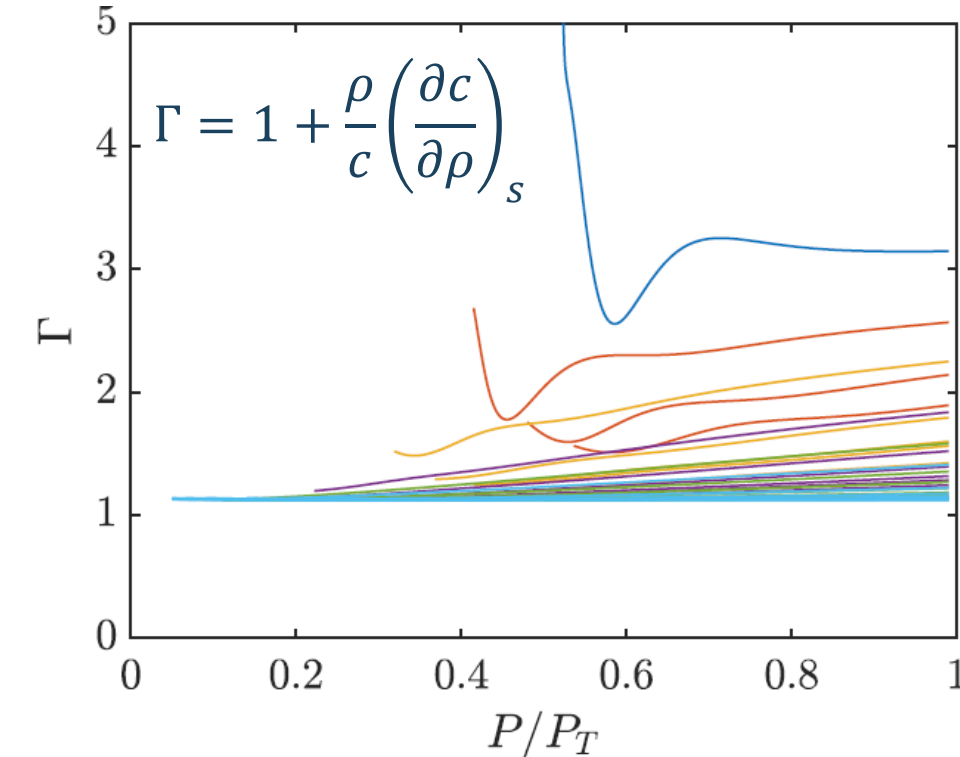
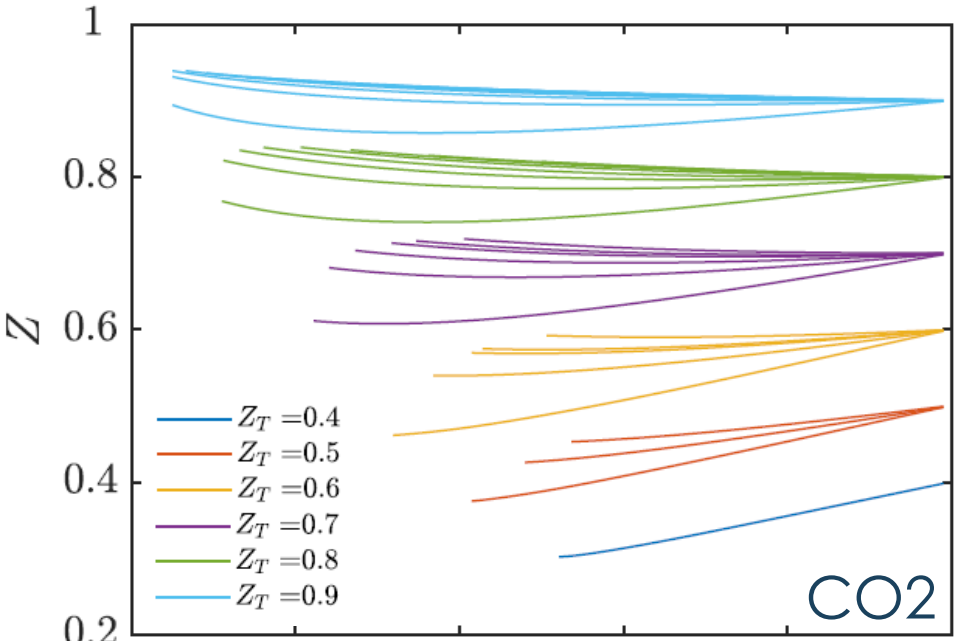


# Results: $Z$ and $\Gamma$ along expansions (LMC – CO<sub>2</sub>)



- Same  $Z_T \rightarrow$  **NOT** similar expansions  
 $\rightarrow$  similar  $Z$ , **NOT** similar  $\Gamma$

# Results: $Z$ and $\Gamma$ along expansions (LMC – CO<sub>2</sub>)



*iso* –  $Z$  // and vertical on Branch A  $\rightarrow$  same  $Z_T$ , similar  $Z \approx const$   
*iso* –  $Z$  **NOT** // *iso* –  $\Gamma$   $\rightarrow$  same  $Z_T$ , **NOT** similar  $\Gamma$  along expansions