Formation of rarefaction shockwaves in non-ideal gases with temperature gradients

Nitish Chandrasekaran

Propulsion & Power Group, TU Delft

NICFD 2020
for Propulsion & Power

October, 29-30
Delft, Netherlands

TU Delft

Propulsion
& Power

27-10-2020

Background

- Acoustic waves in fluids with negative fundamental derivative of gasdynamics Γ exhibit behaviour opposite to that in flows with $\Gamma > 0$

- Finite amplitude acoustic waves in BZT fluids with temperature gradients – not widely investigated[1]

  - For example, in long ducts containing organic fluids in BZT conditions

- Since Γ is sensitive to T, such mediums can impact wave propagation & shock formation significantly

[1] S. Muralidharan and R. Sujith. Shock formation in the presence of entropy gradients in fluids exhibiting mixed nonlinearity. *Physics of Fluids*, 16(11):4121–4128, 2004.

Motivation

- To investigate the effect of T gradient on both the wave propagation and shock formation
- Identify critical parameters to quantify/characterise effect of given T gradient on propagating nonlinear distortions

Simulating a nonlinear distortion

Westervelt Equation[2,3]

$$\underbrace{\rho\nabla.(\rho\nabla p) - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}}_{\text{Linear Wave Equation}} + \underbrace{\frac{\delta}{c_0^4}\frac{\partial^3 p}{\partial t^3}}_{\text{Dissipation term}} + \underbrace{\frac{\Gamma}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2}}_{\text{Nonlinearity term}} = 0$$

$$\Gamma = \beta = 1 + \frac{\rho_0}{2c_0^2}\left(\frac{\partial^2 p}{\partial v^2}\right)_{s,\rho=\rho_0}$$

ρ – Density

p – Pressure

c_0, ρ_0 - sound speed, density @ rest

---

[2] W. Lauterborn, T. Kurz, and I. Akhatov. Nonlinear acoustics in fluids. In *Springer Handbook of Acoustics*, pages 265–314. Springer, 2014.

[3] I. Shevchenko and B. Kaltenbacher. Absorbing boundary conditions for nonlinear acoustics: The westervelt equation. *Journal of Computational Physics*, 302:200–221, 2015.

Numerical Setup

- 2^{nd} order Finite Difference Time Domain (FDTD)[4]
- Domain dimensions – 9 m in length, dx = 1 mm, dt = 1 μs
- Medium properties – D6 @ 9 bar & 369 ^{0}C where $\Gamma = -0.15$ – REFPROP[5,6]

[4] A. Haigh, B. E. Treeby, and E. C. McCreath. Ultrasound simulation on the cell broadband engine using the westervelt equation. In *International Conference on Algorithms and Architectures for Parallel Processing*, pages 241–252. Springer, 2012.

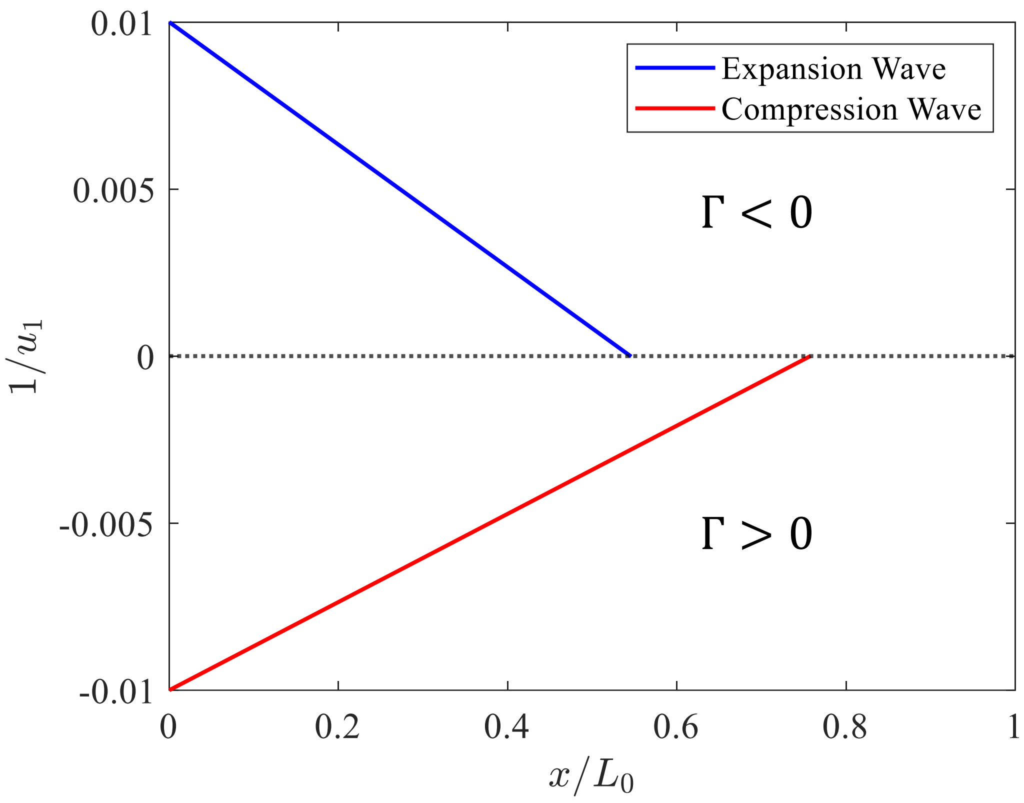
[5] P. Colonna, N. Nannan, and A. Guardone. Multiparameter equations of state for siloxanes:[(ch3) 3-si-o1/2] 2-[o-si-(ch3) 2] i= 1,. . . , 3, and [o-si-(ch3) 2] 6. *Fluid Phase Equilibria*, 263(2):115–130, 2008.

[6] E. W. Lemmon, , I. H. Bell, M. L. Huber, and M. O. McLinden. NIST Standard Reference Database 23: Reference Fluid Thermodynamic and Transport Properties-REFPROP, Version 10.0, National Institute of Standards and Technology, 2018.

Interpreting Results

$$u_1 = \frac{dw}{dx} \text{ where } w = \frac{p}{\rho c}$$

- Slope calculated @ wavefront[1]
- Shock is formed when $\frac{1}{u_1} = 0$

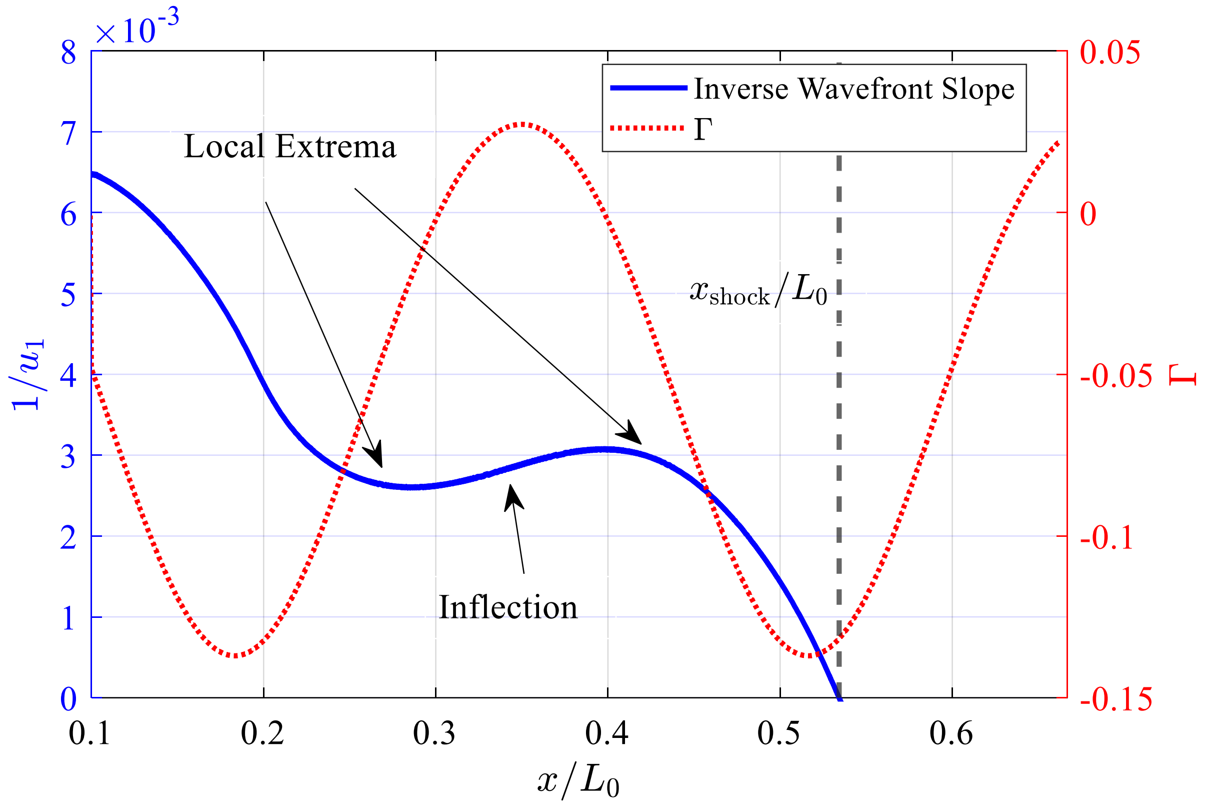


const. T medium

[1] S. Muralidharan and R. Sujith. Shock formation in the presence of entropy gradients in fluids exhibiting mixed nonlinearity. *Physics of Fluids*, 16(11):4121–4128, 2004.

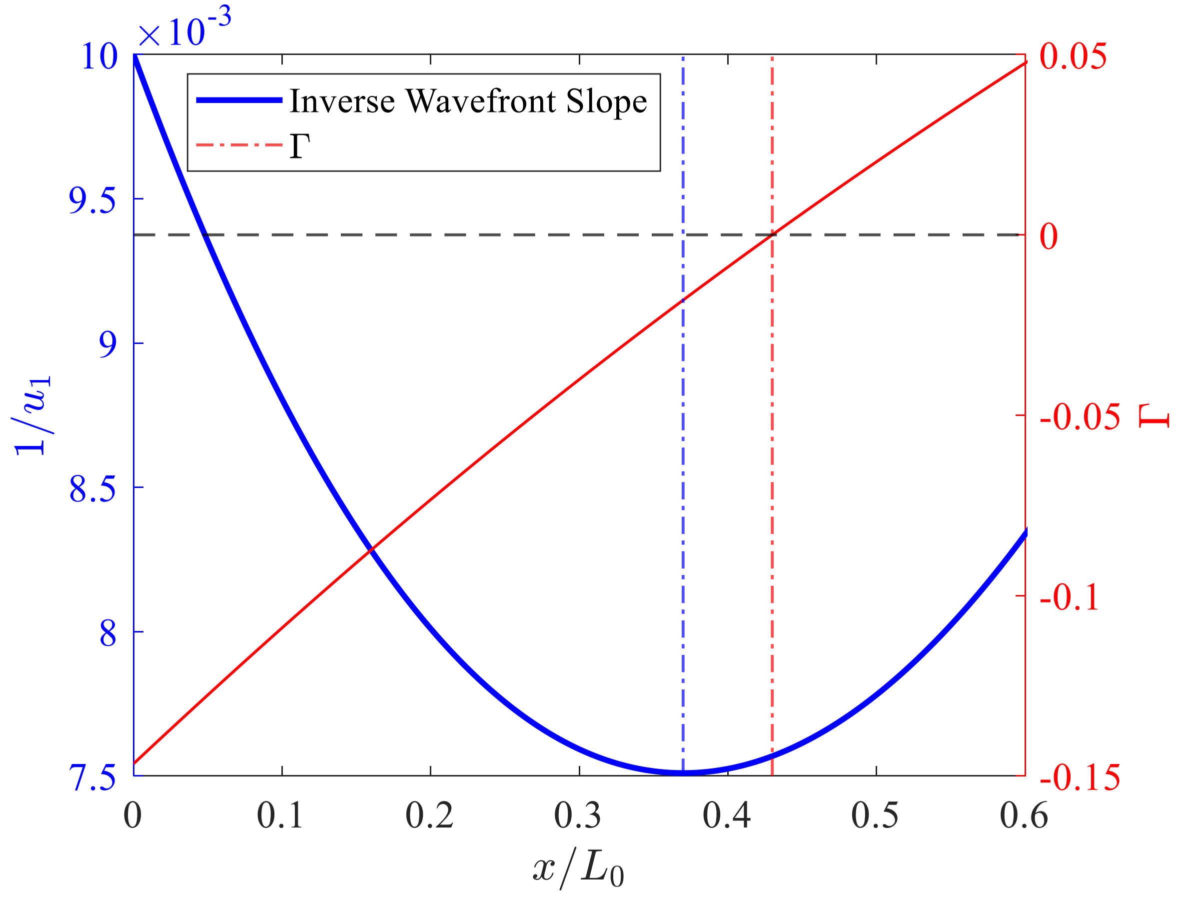
Results & Discussion

Waveslope – medium with sine T variation



- Wave repeatedly steepens and relaxes as T varies
- Inflection in wavefront slope when medium $\Gamma > 0$
- Rarefaction still shocks since bulk of medium is in $\Gamma < 0$

Wave propagation – linearly increasing T



- Linear T increase from 367.5 ^{0}C to 372 ^{0}C
- Compression wave initially relaxes as expected since $\Gamma < 0$
- Wave continues to relax even after $\Gamma > 0$ (between ▬ · · and ▬ · ·)

Factors influencing wave steepening

- Tangent to slope[1]:

$$\cancelto{0}{\frac{d}{dx}\left(\frac{1}{u_1}\right)} = \frac{1}{c}\left(\Gamma + \frac{1}{u_1}\overbrace{\left[\frac{3}{2}\frac{dc}{dx} + \frac{c}{2\rho}\frac{d\rho}{dx}\right]}^{Y}\right)$$

- When the tangent is zero:

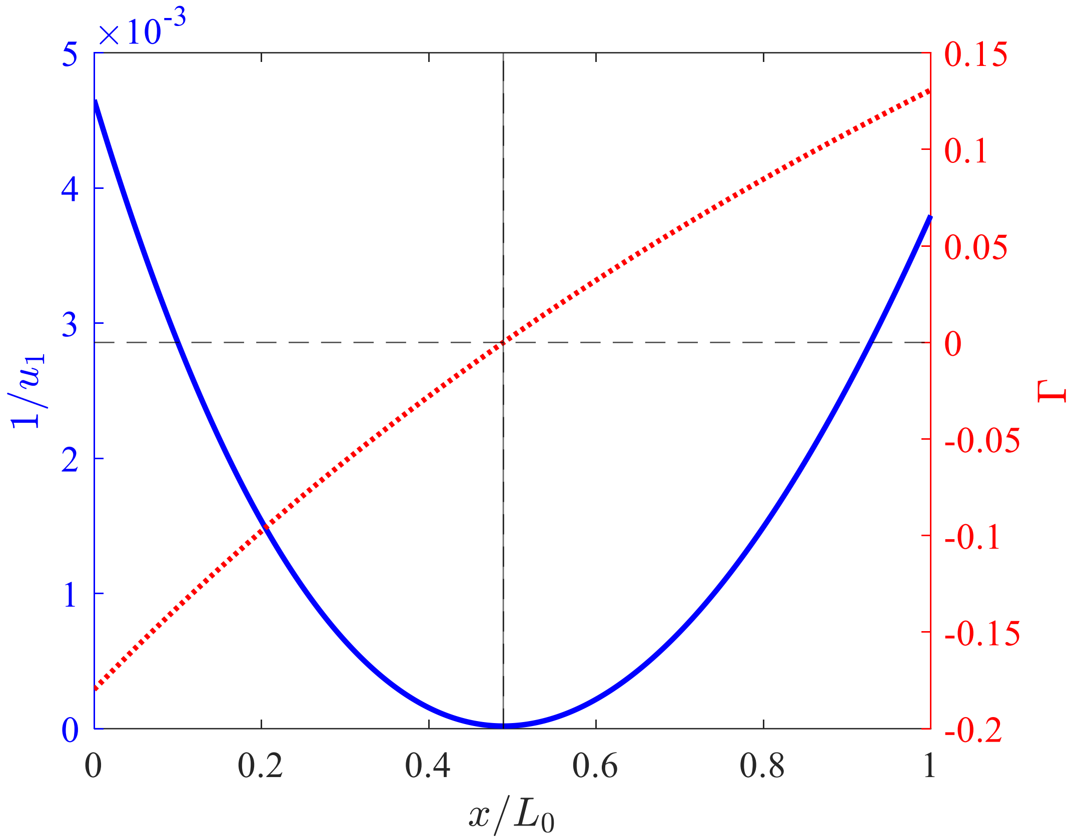
$$\Gamma = -\frac{1}{u_1}Y$$

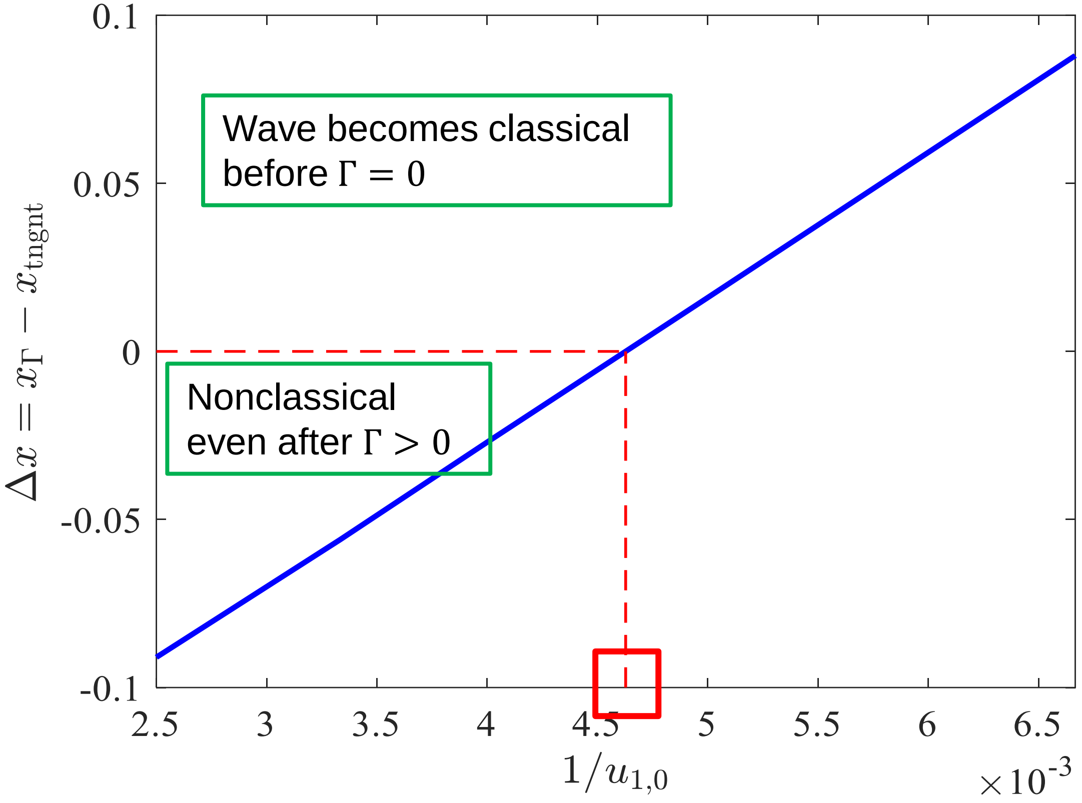
- When Γ is also zero, since $Y \neq 0$ → $1/u_1$ = 0 → special case when the wave just shocks

---

[1] S. Muralidharan and R. Sujith. Shock formation in the presence of entropy gradients in fluids exhibiting mixed nonlinearity. *Physics of Fluids*, 16(11):4121–4128, 2004.

Gap decreases with $1/u_{1,0}$





Wave becomes classical before $\Gamma = 0$

Nonclassical even after $\Gamma > 0$

Conclusions

- Sign and magnitude of Γ sets the general trend for a steepening nonlinear distortion

- Locally, varying sound speed and density has significant impact on behaviour of wave

  - Nonlinear rarefaction waves can relax even when $\Gamma < 0$

  - Vice-versa for compressions

Next steps

- Results show effect of medium temperature gradient on nonlinear wave propagation before shock formation

- How does it affect shock formation?
  - Preliminary results → waves can shock earlier when T varies when compared to homogeneous conditions
  - Ongoing work to analyse and identify relevant parameters linking medium disturbance and the wave properties

Thank You!!