Imperial College London

OIST

Non-ideal gas effects on supersonic-nozzle transfer functions

NICFD2020 - TU Delft

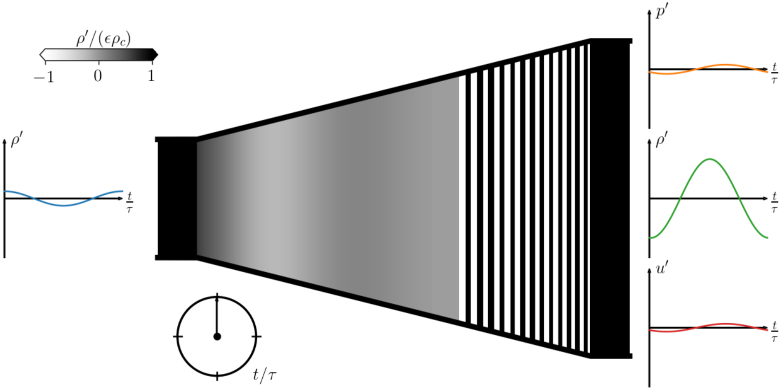
Stephen Winn
sdw16@imperial.ac.uk

Emile Touber
emile.touber@oist.jp

29^{th} October 2020

Outline

1. Linearised quasi-one dimensional nozzle flow
2. Non-ideal gas modelling
3. Isentropic expansion transfer function
4. Shocked-flow transfer function
5. DNS comparison


$\rho'/(\epsilon\rho_c)$
-1
0
1
ρ'
$\frac{t}{\tau}$
t/τ
p'
$\frac{t}{\tau}$
ρ'
$\frac{t}{\tau}$
u'
$\frac{t}{\tau}$

Motivation

- Few real flows are uniform
- Understand frequency response of non-ideal, non-uniform subsonic and supersonic flow
- Useful for many physical applications e.g.

  expansion between turbine blades
  jet engines
  regions of flow that can be modelled as such
- Quasi-1D offers 'simple' framework

Linearised quasi-one dimensional nozzle flow

Governing equations for inviscid, quasi-one dimensional flow:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho u\\ \rho e_t\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\rho u\\ \rho u^2+p\\ u(\rho e_t+p)\end{bmatrix}+\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x}\begin{bmatrix}\rho u\\ \rho u^2\\ u(\rho e_t+p)\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

with the arbitrary gas equation of state (EoS):

$$p=p(\rho,e)\quad\text{where}\quad e=e_t-\frac{u^2}{2}$$

Linearised form obtained by assuming a harmonic perturbation:

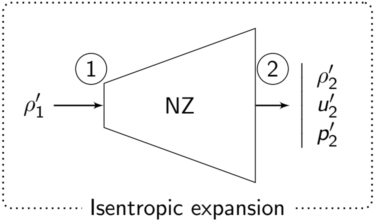
$$q(x,t) = \underbrace{\bar{q}(x)}_{\textit{baseflow}} + \epsilon \underbrace{q(x)'}_{\textit{perturbation}} e^{-\mathrm{i}\omega t} \quad \text{with} \quad \epsilon \ll 1, \quad \omega \in \mathbb{R}_{>0}$$

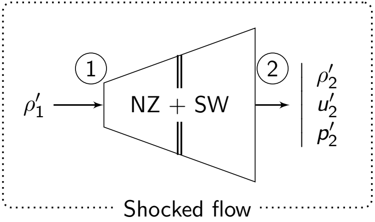
which yields:

$$\mathbf{C}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix} \rho' \\ u' \\ p' \end{bmatrix} = \left(\mathbf{B} + \mathrm{i}\frac{L}{\lambda}\mathbf{I}\right)\begin{bmatrix} \rho' \\ u' \\ p' \end{bmatrix} \tag{1}$$

No assumption made on reduced wavelength (λ/L) i.e. transfer function (TF) dependent on frequency.

Boundary conditions: strictly supersonic inlet with entropy perturbation.





Shocked case: Linearised Rankine–Hugoniot conditions used.

Non-ideal gas modelling

In the paper:

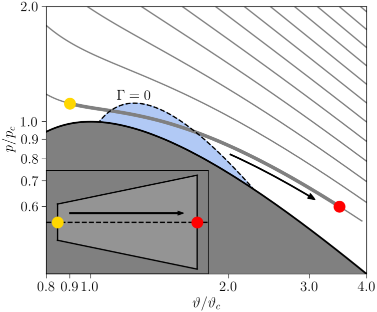
- ideal gas law used as reference
- non-ideal effects illustrated with van der Waals gas law operating close to the critical point

Preliminary results:

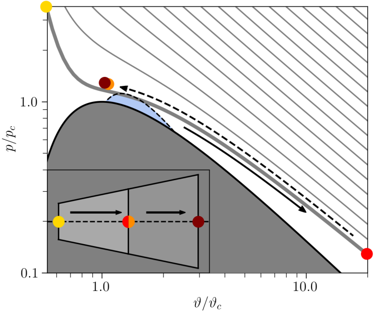
- CoolProp: Span-Wagner multi-parameter EoS for D6

Non-ideal gas modelling

(p-ϑ) space of interest for van der Waals with $\gamma = 1.013$ representative of PP10



Isentropic expansion



Shocked flow

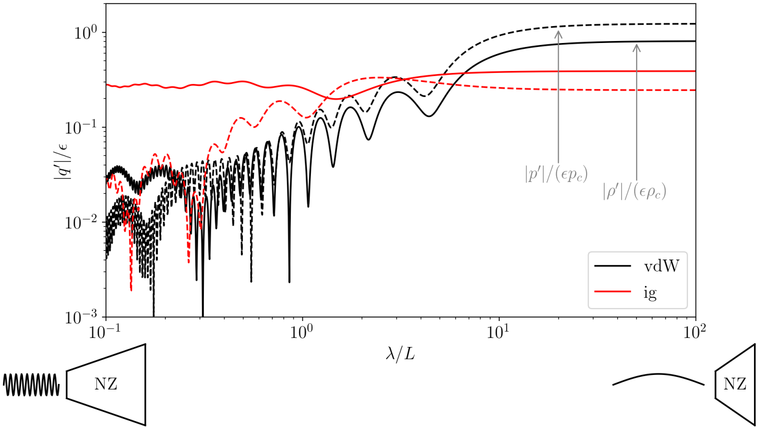
Isentropic expansion transfer function

For fixed geometry and operating range, the TF is dependant on 2 parameters:

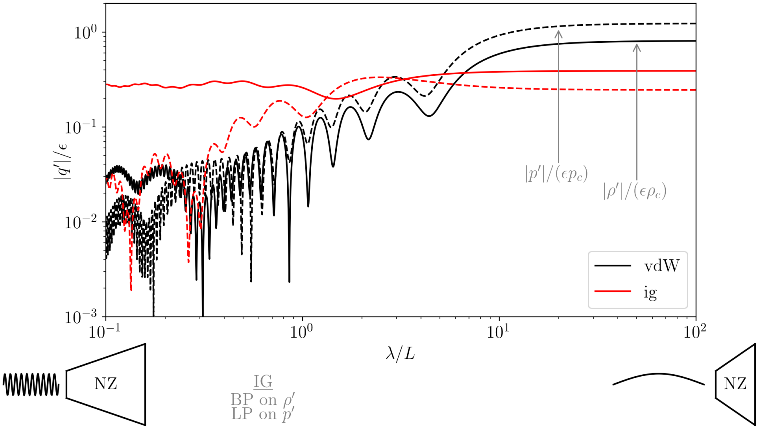
- mass flow $\dot{m}$
- reduced perturbation wavelength λ/L

Scan reduced frequency space and plot transfer function over the chosen λ/L range

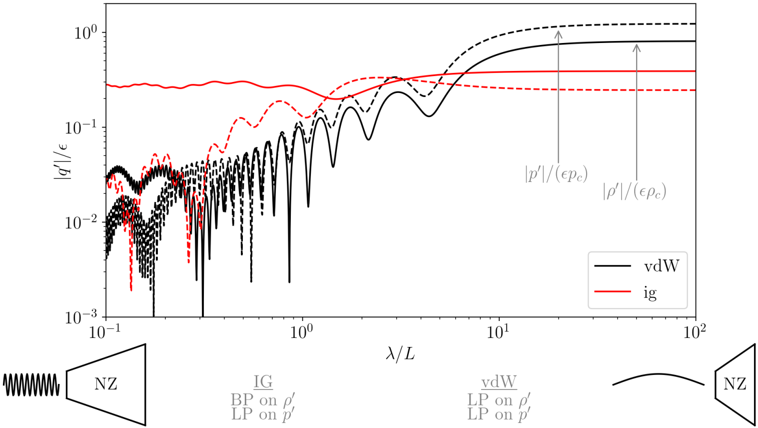
Isentropic expansion transfer function



Isentropic expansion transfer function



Isentropic expansion transfer function



Shocked-flow transfer function

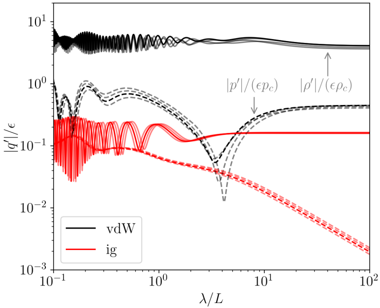
Adding a shock requires specifying two additional conditions:

- outlet thermodynamic condition for the base-flow
- outlet perturbation condition

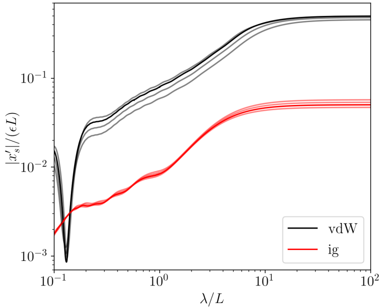
We choose:

- Shock placed at $\bar{x}_s/L = 2/3 \rightarrow$ determines outlet conditions
- Non-reflecting outlet and no forcing

Shocked-flow transfer function



Transfer function



Shock displacement

Along isentrope defined by the point $(p, T) = (0.13p_c, T_c)$.

DNS comparison

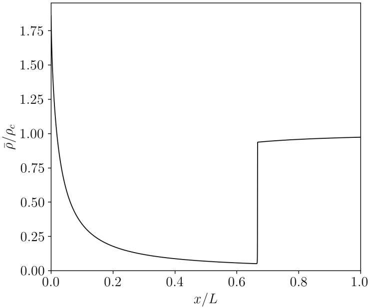
Comparison to time integration of full non-linear equations with linear domain perturbation ($\epsilon \sim 10^{-3}$). Numerics:

- fully explicit (time $\rightarrow$ RK3, space $\rightarrow$ 4^{th} order centred FD)
- arbitrary gas characteristic boundary conditions
- artificial bulk viscosity to capture shock

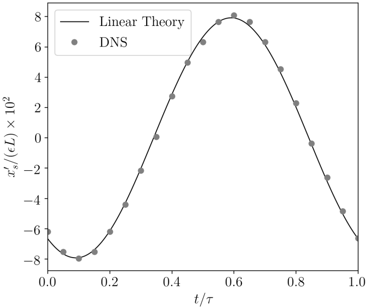
With sufficient resolution to resolve the shock displacement in the linear regime.

DNS comparison

Results for $\lambda/L = 1$, $\epsilon = 4 \times 10^{-3}$ after 10 periods



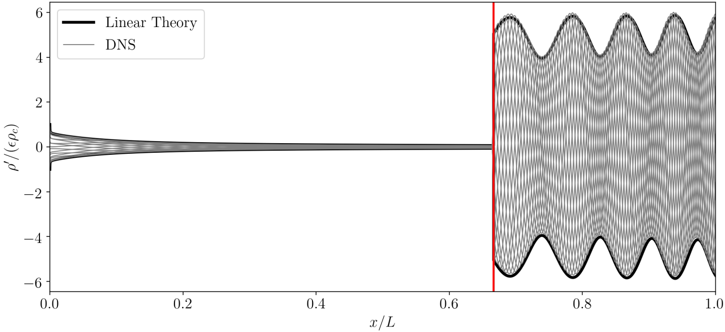
Base-flow density



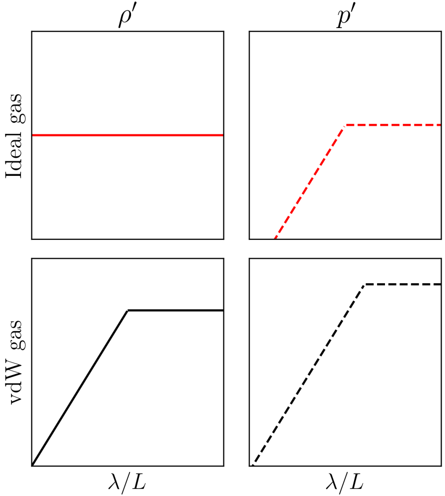
Shock displacement

DNS comparison

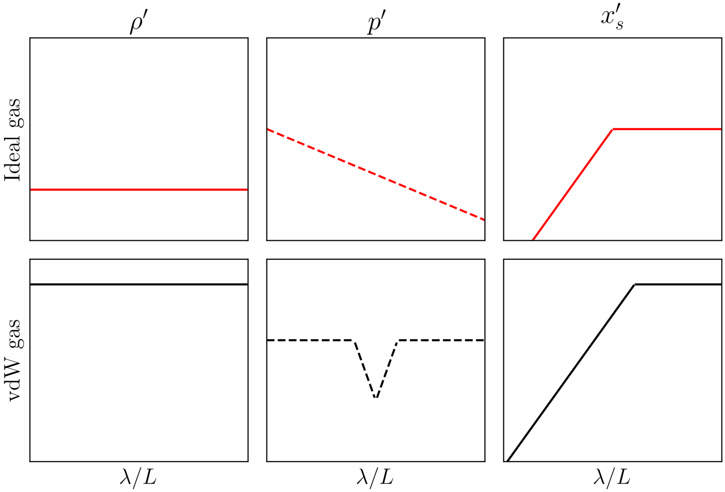
Results for $\lambda/L = 1$, $\epsilon = 4 \times 10^{-3}$ after 10 periods



Summary: isentropic expansion



Summary: shocked flow



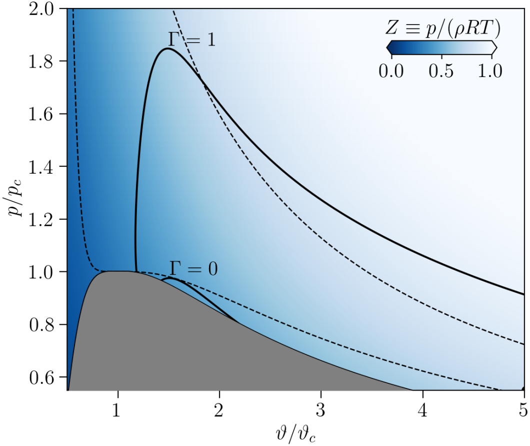
Conclusion

- highlighted non-ideal impact transfer function (‘filter behaviour’)
- shock properties can be exploited (e.g. maximise/minimise a certain fluctuation)
- linear analysis can complement steady-state analysis

Thank you for your attention!
Q&A

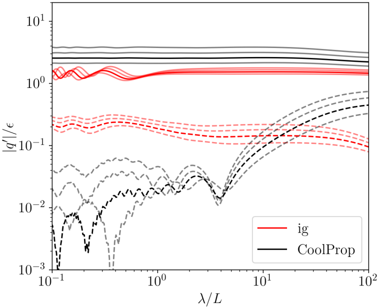
Realistic equation of state

EoS call replaced with CoolProp call to properties of Siloxane D6

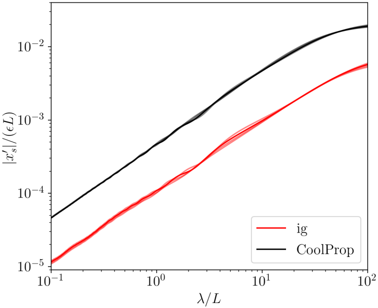


Realistic equation of state

Results for siloxane D6:



Transfer function



Shock displacement

Matrix coefficients for LEE

$$\begin{bmatrix} \bar{u} & \bar{\rho} & 0 \\ 0 & \bar{u} & \dfrac{1}{\bar{\rho}} \\ 0 & \bar{\rho} c^2 & \bar{u} \end{bmatrix} \frac{\mathrm{d}\mathbf{q}'}{\mathrm{d}x} =$$

$$\left(\frac{1}{(\bar{M}^2 - 1)A} \frac{\mathrm{d}A}{\mathrm{d}x} \begin{bmatrix} -\tilde{c}\bar{M}^3 & \bar{\rho} & 0 \\ \dfrac{\bar{M}^2 \tilde{c}^2}{\bar{\rho}} & -\bar{M}\tilde{c} & 0 \\ -\bar{M}^3 \tilde{c}^2 \left(\tilde{c} + 2\bar{\rho}\tilde{c}_\rho\right) & \bar{\rho}\tilde{c}^2 & -2\bar{M}^3 \tilde{c}^2 \tilde{c}_p \bar{\rho} \end{bmatrix} - i\omega \mathbf{I} \right) \mathbf{q}'$$

where $\tilde{c} = c(\bar{\rho}, \bar{p}),\ \tilde{c}_\rho = \left(\partial c / \partial \rho\right)_p (\bar{\rho}, \bar{p}),\ \tilde{c}_p = \left(\partial c / \partial p\right)_\rho (\bar{\rho}, \bar{p})$

Linearised Rankine-Hugoniot equations

$$
\begin{bmatrix} \bar{u}_b & \bar{\rho}_b & 0 \\ \bar{u}_b^2 & 2\bar{\rho}_b\bar{u}_b & 1 \\ -\left.\frac{\partial \wp}{\partial \rho}\right|_b & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho_b' \\ u_b' \\ p_b' \end{bmatrix}
+ \begin{bmatrix} -i\omega[\![\bar{\rho}]\!] \\ \frac{\bar{\rho}_a^2\bar{u}_a^2}{A}\frac{\mathrm{d}A}{\mathrm{d}x}\left[\!\left[\frac{1}{\bar{\rho}}\right]\!\right] \\ \left.\left(\frac{\partial \wp}{\partial p}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} + \frac{\partial \wp}{\partial \rho}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}x}\right)\right|_a + \left.\left(\frac{\partial \wp}{\partial \rho}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}x} - \frac{\mathrm{d}\bar{p}}{\mathrm{d}x}\right)\right|_b \end{bmatrix} x_s' =
\begin{bmatrix} \bar{u}_a & \bar{\rho}_a & 0 \\ \bar{u}_a^2 & 2\bar{\rho}_a\bar{u}_a & 1 \\ \left.\frac{\partial \wp}{\partial \rho}\right|_a & 0 & \left.\frac{\partial \wp}{\partial p}\right|_a \end{bmatrix} \begin{bmatrix} \rho_a' \\ u_a' \\ p_a' \end{bmatrix}
$$