Mathematical Programming for the Market Integration of Power-to-X Hybrid Power Plants
The Case of a Plant in Denmark

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HOMEY project
Towards $H_2$-driven business model: Portfolio management of a multi-energy system under uncertainty
Outline

Preliminaries
  - Why Power-to-X
  - An Overview of TSO Services
  - Challenges

Physical Modeling of Electrolyzers

Value of Ancillary Services for Electrolyzers

Portfolio Management Problem: Distributionally Robust Optimization
Preliminaries:

Why Power-to-X?
P2X strategy in Denmark: New electrolyzers to be installed

- 4-6 GW of electrolyzers by 2030
- 1.25 billion DKK in support
- CO₂ emission reduction: 2.5-4.0 million tons
Why Power-to-X?

1. Current **hydrogen demand**
   - **90 Mt** of H2 in 2020
   - Only **0.03%** from water electrolysis

2. **Indirect electrification** and decarbonization of
   - Heavy **transport** (heavy-duty vehicles, aviation and shipping)
   - **Industry**

3. Added value to the **power system**:
   - Higher **utilization** of renewables in the grid (reduce curtailment)
   - **Ancillary services** provision (high ramping rates)

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Source: IEA, 2021
Hybrid Power Plant

What are the products of the hybrid power plant?
Hybrid Power Plant

Portfolio of products:

- **Power** (to be sold in various trading floors)
- **Hydrogen** (fixed price)
- **TSO services** such as frequency-based ancillary services (electrolyzers are fast flexible demands!)
- **DSO services** (if the plant is connected to a distribution grid) such as grid congestion services
Preliminaries:

An Overview of TSO Services
Recall: Who clears what market (European setup)?

By the market operator (e.g., Nord Pool)

- Day-ahead market
- Intra-day market

By the TSO (e.g., Energinet in Denmark, or Statnett in Norway)

- TSO services (reservation)
- Balancing market

Do these contracts ensure safe operation of the system?

- Yes
- No?

Reserve capacities booked to be activated later (if needed) in the balancing stage

Real-time operation (Actual time for the physical delivery of energy)
Synchronous Grid Areas in Europe
Energinet is operating the Danish power system in two areas → different ancillary services exist in DK1 and DK2.
**Frequency-based Ancillary Services in DK1 and DK2**

**Ancillary services in DK1** (as part of the continental area)
- FCR (frequency containment reserve)
- aFRR (automatic frequency restoration reserve)
- mFRR (manual frequency restoration reserve)

**Ancillary services in DK2** (as part of the Nordic area)
- FFR (fast frequency reserve)
- FCR-D (D stands for disturbance)
- FCR-N (N stands for normal)
- aFRR
- mFRR

Source: Energinet (Gennemgang af Nuvaerende Systemydelse Markeder)
Specifics of Ancillary Services in DK1 and DK2

Source: Energinet (Gennemgang af Nuværende Systemydelse Markeder)
Potential Service Providers in DK1 and DK2

Source: Energinet (Gennemgang af Nuværende Systemydelse Markeder)
Potential Service Providers in DK1 and DK2

Hybrid power plants can provide all services.

Source: Energinet (Gennemgang af Nuværende Systemydelser Markeder)
Ancillary Service Markets in DK1 and DK2

<table>
<thead>
<tr>
<th></th>
<th>aFRR/ mFRR</th>
<th>FCR-N/ FCR-D</th>
<th>FCR</th>
<th>mFRR</th>
<th>FFR</th>
<th>FCR-N/ FCR-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>⌧</td>
<td>PaB/UP</td>
<td>PaB</td>
<td>UP</td>
<td>UP</td>
<td>UP</td>
<td>PaB</td>
</tr>
<tr>
<td>$</td>
<td>R+A</td>
<td>R+A/R</td>
<td>R</td>
<td>R+A</td>
<td>R</td>
<td>R+A/R</td>
</tr>
<tr>
<td>⌚</td>
<td>5-15m/15m</td>
<td>150s/5-30s</td>
<td>15-30s</td>
<td>15m</td>
<td>1s</td>
<td>150s/5-30s</td>
</tr>
</tbody>
</table>

Timeline for reservation auctions for all ancillary services in Denmark. Service mFRR has monthly and daily auctions. Both FCR-N and FCR-D have two auctions, running two days and one day before delivery. The auction scheme is shown together with the origin of payment (third row) and the speed of full response (fourth row). For aFRR, the speed of full response is 15 min in DK1 and 5 minutes in DK2, and will be 5 min in DK1 from 2024. **Abbreviations:** PaB: pay-as-bid. UP: uniform pricing. R: reservation payment. A: activation payment. M: month. D: day. m: minutes. s: seconds.

**Source:** Peter A. V. Gade, Trygve Skjøtskift, Henrik W. Bindner, and JK, "Ecosystem for demand-side flexibility revisited: The Danish solution", *The Electricity Journal*, vol. 35, no. 9, Article no. 107206, November 2022 [link][arXiv]
Historical data: Activated FCR-D and FCR-N in DK2 (2021-2022)

FCR-N (when the frequency was between 49.9 and 50 Hz)

FCR-N (when the frequency was between 50 and 50.1 Hz)

FCR-D UP (when the frequency was between 49.5 and 49.9 Hz)

FCR-D Down (when the frequency was between 50.1 and 50.5 Hz)

Credit: Marco Saretta
Historical data: Activated FCR-D and FCR-N in DK2 (2021-2022)

FCR-D was very rarely activated! Service providers received payments due to capacity reservation but were activated very rarely!

Credit: Marco Saretta
Historical data: FCR-D and FCR-N prices in DK2 (2015-2022)

FCR-D Down (a service when frequency is between 50.1 and 50.5 Hz) started in January 2022

Credit: Marco Saretta
Challenges
Portfolio Management Problem of the Hybrid Power Plant

**Input data**

**Electricity**
- Day-ahead price forecast
- Balancing price forecast
- Wind power forecast

**Ancillary services**
- FCR/aFRR/mFRR reservation price forecast
- Forecast of activation for each service

**Hydrogen**
- Hydrogen price
- Minimum hydrogen demand (e.g., over a day)
- Tube trailer availability (e.g., over the next day)

**Other technical data**

Maximize (expected) profit
Subject to physical and bidding constraints

Scheduling (bidding) decisions
Portfolio Management Problem of the Hybrid Power Plant

Maximize (expected) profit
Subject to physical and bidding constraints

Input data

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- **Other technical data (certain)**

**Maximize (expected) profit**

Subject to physical and bidding constraints

**Scheduling (bidding) decisions**

Any challenge?
Portfolio Management Problem

Maximize (expected) profit
Subject to physical and bidding constraints

How to properly model uncertainty?

High dimensionality
- Many (potentially correlated) sources of uncertainty

Non-stationarity
- The analysis of historical data suggests our stochastic environment is not necessarily stationary!

Lack of enough historical data
- Example 1: FCR-D down market has been recently established
- Example 2: Denmark has recently switched to a 1-price balancing scheme (before it was 2-price)

Conditionality
- The distribution of forecast error depends on the point forecast.

Electricity (uncertain)
- Day-ahead price forecast
- Balancing price forecast
- Wind power forecast

Ancillary services (uncertain)
- FCR/aFRR/mFRR reservation price forecast
- Forecast of activation for each service

Hydrogen
- Hydrogen price (fixed price)
- Minimum hydrogen demand (e.g., over a day) (certain)
- Tube trailer availability (e.g., over the next day) (uncertain)

Other technical data (certain)

Input data

etc
Portfolio Management Problem of the Hybrid Power Plant

Maximize (expected) profit
Subject to physical and bidding constraints

Input data

Electricity (uncertain)
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Hydrogen
- Hydrogen price (fixed price)
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Other technical data (certain)

Any other challenge related to modeling the physics of the plant?
Portfolio Management Problem of the Hybrid Power Plant

Maximize (expected) profit
Subject to physical and bidding constraints

- **Electricity (uncertain)**
  - Day-ahead price forecast
  - Balancing price forecast
  - Wind power forecast

- **Ancillary services (uncertain)**
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- **Hydrogen**
  - Hydrogen price (fixed price)
  - Minimum hydrogen demand (e.g., over a day) (certain)
  - Tube trailer availability (e.g., over the next day) (uncertain)

- **Other technical data (certain)**

Electrolyzers are non-linear assets:
Several stakeholders in DK are interested in different aspects of such a portfolio management problem.
Physical Modeling of Electrolyzers:
Piece-wise Linear Approximation

*Question:* To what extent does it matter to model the physics of electrolyzers in a detailed manner?


Codes: [https://github.com/mtba-dtu/detailed-electrolyzer-model](https://github.com/mtba-dtu/detailed-electrolyzer-model)
Hydrogen Production and Efficiency Curves

Consider a 52.25-MW alkaline electrolyzer, working at 90 °C and 30 bar.

A simple idea:
- Hydrogen production curve is linearized by a couple of red segments (each segment: $Ax+B$) → resulting model: MILP
- The approximate efficiency curve (red) is still nonlinear due to the intercept $B$, but this curve will not be used in our optimization model.
Operating States of Electrolyzers

**Online**
- Power consumption active
- Hydrogen production

**Offline**
- No power consumption
- No hydrogen production
- When transitions from Offline to Online, pay the cold startup cost

**Standby**
- No hydrogen production
- Power consumption reduced, to keep the electrolyzer hot and pressurized
- No cold start cost when transitions from Standby to Online
Case Study

- Only electricity and hydrogen (no ancillary services)
- One year simulation (8760 hours)
- Deterministic (known wind power and price profiles)
Case Study

- Only electricity and hydrogen (no ancillary services)
- One year simulation (8760 hours)
- Deterministic (known wind power and price profiles)

### Wind farm
- Capacity: 104.5 MW
- Standby load: 52.25 MW
- Minimum load: 7.838 MW
- Pressure: 30 bar
- Temperature: 90 °C
- Maximum current density: 5,000 A/m²
- Startup cost: 50 €/MW
- TSO tariff: 15.06 €/MWh

### Electrolyzer
- Capacity: 22,000 kg
- Input/Output: 912.13 kg/h
- Inlet temperature: 40 °C
- Inlet pressure: 30 bar
- Outlet pressure: 200 bar
- Mechanical efficiency: 75%

### Storage
- Price: 2.10 €/kg
- Minimum demand: 2,750 kg/d
Results: Impact of the number of segments 1/2
(Number of states fixed)

By adding more segments, the electrolyzer consumes power more **dynamically** (following the day-ahead price signal)

This happens within a specific **price range**

---

Power consumption schedule of the electrolyzer in an example high-wind day
Results: Impact of the number of segments 2/2

Histogram of the day-ahead hourly prices in 2019

\[\lambda^{DA,ub} = \frac{\lambda^h\eta_{\max} P_{\eta_{\max}}}{P_{\eta_{\max}} - P_{sb}}.\]

\[\lambda^{DA,lb} = \lambda^h \left( \eta^f + C^e \eta' (x) \right)_{x=C^e} \]

\(\lambda^{DA,ub}\) and \(\lambda^{DA,lb}\) depend only on:
- electrolyzer efficiency curve
- standby power consumption
- hydrogen price

Based on the range of day-ahead prices, we know “a priori” whether adding more segments matter.
Physical Modeling of Electrolyzers:
Conic Relaxation


Codes: [https://github.com/ELMA-Github/conic_electrolyzer_paper](https://github.com/ELMA-Github/conic_electrolyzer_paper)
Potential Solutions for Convexification of the Curve

Hydrogen production curves (convexified and then to be used in the model)

Efficiency curves (not to be used in the model)
Potential Solutions for Convexification of the Curve

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Efficiency curves (not to be used in the model)

Model HYP-MIL
- Piece-wise linear approximation of the hydrogen production curve
- One binary per segment
Potential Solutions for Convexification of the Curve

Model HYP-L
- Outer approximation (relaxation)
- This is a common approach to convexify the gas flow Weymouth equations.
- No binary
Potential Solutions for Convexification of the Curve

Model HYP-SOC
- Fitting a conic equation, with the same efficiency peak
- Conic relaxation
- No binary
Potential Solutions for Convexification of the Curve

Model HYP-MISO

- Fitting two conic equations
- Conic relaxation
- One binary only to choose the active conic constraint
# Case Study: Tightness of the Relaxation

**HYP-SOC model (identical observations for other models)**

<table>
<thead>
<tr>
<th>Day</th>
<th>Minimum daily hydrogen demand constraint</th>
<th>Electricity prices</th>
<th>Wind</th>
<th>Tightness</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Not binding</td>
<td>Most of hours: negative prices</td>
<td>Low wind</td>
<td>Exact</td>
</tr>
<tr>
<td>b</td>
<td>Binding</td>
<td>All hours: positive prices</td>
<td>High wind</td>
<td>Exact</td>
</tr>
<tr>
<td>c</td>
<td>Binding</td>
<td>Most of hours: negative prices</td>
<td>High wind</td>
<td>Inexact</td>
</tr>
<tr>
<td>d</td>
<td>Binding</td>
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<td>High wind</td>
<td>Exact</td>
</tr>
</tbody>
</table>

This is a real case study (17/03/2019)
Case Study: Tightness of the Relaxation

Some observations

➢ **Day (a):** if the demand constraint is not binding, the relaxation is always **exact.**

➢ **Day (b):** if the demand constraint is binding but prices are positive, the relaxation is always **exact.**

➢ **Day (c):** if the demand constraint is binding and there are many negative prices → relaxation is **inexact.**

➢ **Day (d):** if the demand constraint is binding and there are a few negative prices → the relaxation is **exact.**

➢ **Mathematical proofs of the exactness of the conic relaxation under (prevalent) operational circumstances are available in the paper.**
Value of Ancillary Services for Electrolyzers


Codes: https://github.com/marco-srtt/electrolyzer_nordic_FCR

A. Gloppen Johnsen, L. Mitridati, D. Zarrilli, and JK, “Value of ancillary services for electrolyzers,” soon to be shared

Codes: TBD
Hybrid Power Plant in Ancillary Service Markets

Let’s consider a subset of existing services in DK1, including mFRR (up and down) and FCR. There are minimum bid size and bid time length constraints.
Hybrid Power Plant in Ancillary Service Markets

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mFRR up:
- The hybrid power plant is paid for the reservation sold based on mFRR up market price.
- If activated (partial or full), the plant is paid at the balancing price for the energy not consumed. Meanwhile, the plant will produce less hydrogen due to less electric energy consumption.
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- If activated (partial or full), the plant is paid at the balancing price for the energy not consumed. Meanwhile, the plant will produce less hydrogen due to less electric energy consumption.

**mFRR down:**
- The hybrid power plant is paid for the reservation sold based on mFRR down market price.
- If activated (partial or full), the plant pays at the balancing price for the additional energy consumed. Meanwhile, the plant will produce more hydrogen due to extra electric energy consumption.
Hybrid Power Plant in Ancillary Service Markets

Let’s consider a subset of existing services in DK1, including mFRR (up and down) and FCR. There are minimum bid size and bid time length constraints.

mFRR up:
- The hybrid power plant is paid for the reservation sold based on mFRR up market price.
- If activated (partial or full), the plant is paid at the balancing price for the energy not consumed. Meanwhile, the plant will produce less hydrogen due to less electric energy consumption.

mFRR down:
- The hybrid power plant is paid for the reservation sold based on mFRR down market price.
- If activated (partial or full), the plant pays at the balancing price for the additional energy consumed. Meanwhile, the plant will produce more hydrogen due to extra electric energy consumption.

FCR:
- The hybrid power plant is paid for the reservation sold based on FCR market price.
- No payment/cost in the balancing stage for the activation. Not an energy-intense service. The activation is automatic based on frequency deviations between 49.9 and 50.1 Hz, which is quite symmetric over a time period, so one can imagine FCR up and down activations cancel out each other.
Hybrid Power Plant in Ancillary Service Markets

- We use realized prices in DK1 in 2021 and 2022 (ideal benchmark to obtain the highest potential value of services).
- Deterministic model
Hybrid Power Plant in Ancillary Service Markets

Revenues

Prices

α = 1  α = 0  Oracle  No AS  Model version

DK1 2021

DK1 2022

H2  DA  FCR  mFRR up  Bal

Profit
In DK1 (2021), an electrolyzer had a maximum potential to increase its profit by by 50% providing FCR and mFRR services!
In DK1 (2022), there was a HUGE potential!
Hybrid Power Plant in Ancillary Service Markets

Quantities

Ambient"

Reserves [GW]

DK1 2021

DNK 2022

0.1

Model version

α = 1

α = 0

Oracle

No AS

FCR

mFRR up

Prices

Frequency (log)

0

10^3

10^1

0

1000

2000

0

Price (EUR)

Price (EUR)

DK1 2021

DK1 2022
Portfolio Management Problem: Distributionally Robust Optimization
Concept

Maximize profit (expected or risk-averse)

- Electricity price distribution
- Wind generation distribution
- Hydrogen price

Optimal production schedule
Maximize profit (expected or risk-averse)

Optimal production schedule

Ancillary service market prices (FCR, aFRR, mFRR)

Timeline of clearing ancillary service markets in Denmark

Concept

Main challenges:
• Model uncertainty sources accurately
• Model and mitigate risk of decisions

Electricity price distribution
Wind generation distribution
Hydrogen price

Maximize profit (expected or risk-averse)

Optimal production schedule
Concept

D-100

Today

D+1

Historical data

Wind Production

Electricity Price
Concept

Historical forecasting error
= Forecast – Observation
Concept

Historical data

Wind Production Error

Electricity Price Error

Historical forecasting error

= Forecast – Observation
Concept

Historical data
- Wind Production Error
- Electricity Price Error

New deterministic forecasts
- Wind Production
- Electricity Price

Predict forecasting errors
- Historical Wind Forecasting Error
- Historical Price Forecasting Error

Today

Historical forecasting error = Forecast – Observation

Predict tomorrow’s forecasting error
Concept

Scenario-Based Stochastic Optimization

- **Aim:** Max. expected profit for **given distribution**
- **Challenge:** **Data-intensive** to estimate probability distribution $\mathbb{P}$
- **Aim:** Max. expected profit for *given distribution*

- **Challenge:** *Data-intensive* to estimate probability distribution $\mathbb{P}$

---

- **Aim:** Max. expected profit for *worst-case distribution* in ambiguity set

- **Benefit:** *Data-driven* approach
Concept

Main challenges:
• Model **uncertainty** sources accurately
• Model and **mitigate risk** of decisions
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• Model and **mitigate risk** of decisions

Consider **expectation** from **worst-case** forecasting error distribution.
Out-of-Sample Results

![Graph showing out-of-sample results with various lines indicating different strategies, such as DRO, Control, SAA, DET, active realization, and passive realization. The graph plots average daily return against percentage improvement.]

- **DRO**
- **Control**
- **SAA**
- **DET**
- **Active realization**
- **Passive realization**

**Passive Active Gap**
Out-of-Sample Results

Forecast error reducing (less uncertainty)
Out-of-Sample Results

- **Active model (upper bound):** Hydrogen production can be fully adjusted in real time.
- **Passive model (upper bound):** Hydrogen production is fixed to the day-ahead schedule.

Wasserstein DRO
Out-of-Sample Results

Deterministic

Unaware of historical forecast error data
Out-of-Sample Results

- Sample Average Approximation (SAA)
- Wasserstein DRO with adaptive radius
Takeaways

- **Operational details** of P2X assets are important for the scheduling of the system.

- This is a **multi-market** decision-making problem:
  - Day-ahead and balancing electricity markets
  - Ancillary service (FCR, aFRR, mFRR) markets
  - Hydrogen

- Modeling **uncertainty** is key, while there might be lack of sufficient historical data to accurately characterize underlying probability distributions!
Thank you!

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