ETH zürich TU Delft DCSC, 3mE

Gather-and-broadcast frequency control in power systems

Florian Dörfler **Sergio Grammatico**

TU Delft – Power Web seminar

Delft University of Technology

Delft, The Netherlands, Nov 8, 2018

A few (of many) game changers in power system operation

synchronous generator

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$\rightsquigarrow$ power electronics

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scaling

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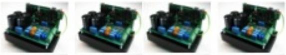
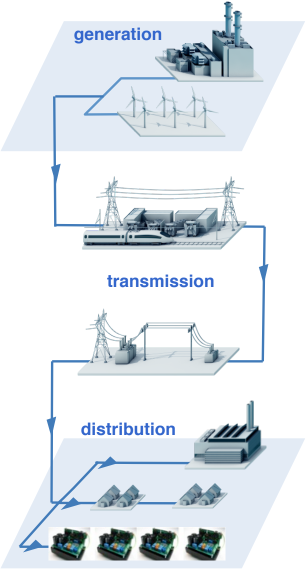


distributed generation



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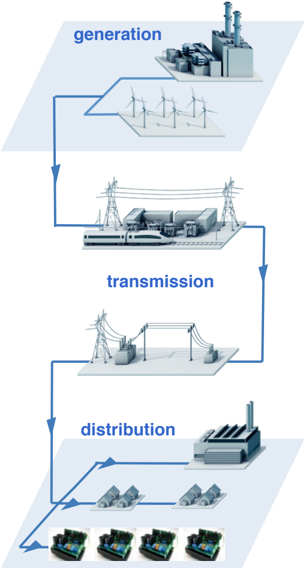
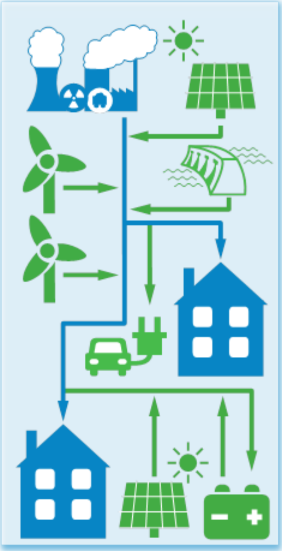
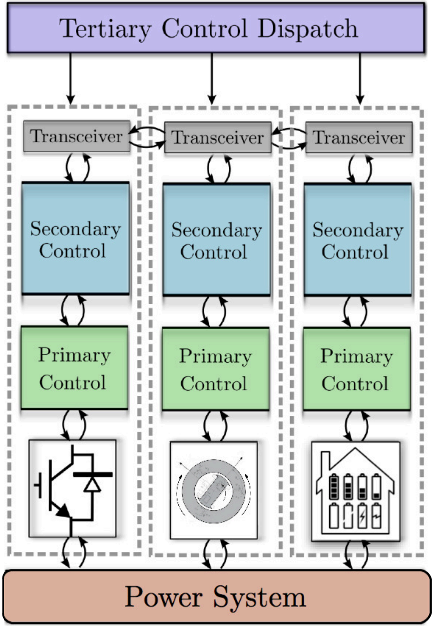


other paradigm shifts



Conventional frequency control hierarchy



3. **Tertiary control** (offline)
   - goal: optimize operation
   - architecture: centralized & forecast
   - strategy: scheduling (OPF)

2. **Secondary control** (slower)
   - goal: maintain operating point
   - architecture: centralized
   - strategy: I-control (AGC)

1. **Primary control** (fast)
   - goal: stabilization & load sharing
   - architecture: decentralized
   - strategy: P-control (droop)

Is this **top-to-bottom architecture** based on **bulk generation control** still appropriate in tomorrow's grid?

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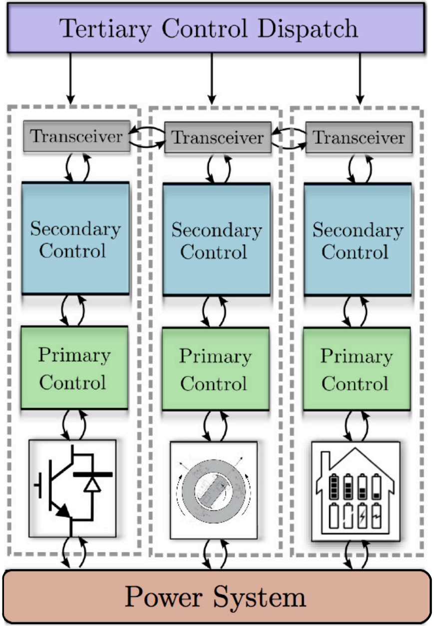

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Outline

Nonlinear differential-algebraic power system model

- generator **swing equations** $i \in \mathcal{G}$

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i + u_i - \sum_{j \in \mathcal{V}} B_{i,j} \sin(\theta_i - \theta_j)$$

- frequency-responsive loads & grid-forming **inverters** $i \in \mathcal{F}$

$$D_i \dot{\theta}_i = P_i + u_i - \sum_{j \in \mathcal{V}} B_{i,j} \sin(\theta_i - \theta_j)$$

- **load buses** with demand response $i \in \mathcal{P}$

$$0 = P_i + u_i - \sum_{j \in \mathcal{V}} B_{i,j} \sin(\theta_i - \theta_j)$$

- $D_i \dot{\theta}_i$ is **primary droop control** (not focus today)
- $u_i \in \mathcal{U}_i = [\underline{u}_i \, , \, \overline{u}_i]$ is **secondary control** (can be $\mathcal{U}_i = \{0\}$)

$\Rightarrow$ **sync frequency** $\omega_{\text{sync}} \sim \sum_i P_i + u_i$ = imbalance

$\Rightarrow$ $\exists$ synchronous equilibrium iff $\sum_i P_i + u_i = 0$ (load = generation)

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Economically efficient secondary frequency regulation

Problem I: frequency regulation

Control $\{u_i \in \mathcal{U}_i\}_i$ to **balance** load & generation: $\sum_i P_i + u_i = 0$

Problem II: optimal economic dispatch

Control $\{u_i \in \mathcal{U}_i\}_i$ to **minimize** the aggregate operational cost:

$$\min_{u \in \mathcal{U}} \ \sum_i J_i(u_i)$$
$$\text{s.t.} \ \sum_i P_i + u_i = 0$$

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$\implies$ identical marginal costs at optimality: $J_i'(u_i^\star) = J_j'(u_j^\star) \ \forall i, j$

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$\Longrightarrow$ identical marginal costs at optimality: $J_i'(u_i^\star) = J_j'(u_j^\star) \ \forall i, j$

Standing assumptions

- **feasibility:** $-\sum_i P_i \in \sum_i \mathcal{U}_i = \sum_i [\underline{u}_i \, , \overline{u}_i]$
- **regularity:** $\{J_i : \mathcal{U}_i \to \mathbb{R}\}_i$ strictly convex & cont. differentiable

critical review of secondary control architectures

Centralized automatic generation control (AGC)

integrate single measurement & broadcast

$$k\,\dot{\lambda} = -\omega_{i^*}$$
$$u_i = \frac{1}{A_i}\lambda$$

☺ inverse optimal dispatch for $J_i(u_i) = \frac{1}{2}A_i u_i^2$

☺ few communication requirements (broadcast)

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☺ few communication requirements (broadcast)

☹ single authority & point of failure $\Longrightarrow$ not suited for distributed gen

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Decentralized frequency control

integrate local measurement

$$k_i \, \dot{\lambda}_i = -\omega_i$$
$$u_i = \lambda_i$$

- ☺ nominal stability guarantee
- ☺ no communication requirements

M. Andreasson, D. Dimarogonas, H. Sandberg, and K. Johansson, "Distributed PI-control with applications to power systems frequency control," in *American Control Conference*, 2014.

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Decentralized frequency control

integrate local measurement

$$k_i \, \dot{\lambda}_i = -\omega_i$$
$$u_i = \lambda_i$$

- ☺ nominal stability guarantee
- ☺ no communication requirements
- ☹ does not achieve economic efficiency
- ☹ $\exists$ biased measurement $\implies$ instability

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Distributed averaging frequency control

integrate local measurement & average marginal costs

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\begin{aligned}
k_i\,\dot{\lambda}_i &= -\omega_i + \sum\nolimits_j w_{i,j}\left(J_i'(u_i) - J_j'(u_j)\right)\\
u_i &= \lambda_i
\end{aligned}
$$

- ☺ stability & robustness certificates
- ☺ asymptotically optimal dispatch

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☺ stability & robustness certificates

☺ asymptotically optimal dispatch

☹ high communication requirements & vulnerable to cheating

☹ utility concern: "give power out of our hands"

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another (possibly better?) control protocol for distributed generation

Motivation: from social welfare to competitive markets

Social welfare dispatch

$$\min_{u \in \mathcal{U}} \ \sum_i J_i(u_i)$$

$$\text{s.t.} \ \sum_i P_i + u_i = 0$$

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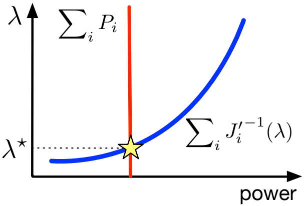

Competitive spot market

1. given a prize λ, player i **bids**

$$u_i^\star = \underset{u_i \in \mathcal{U}_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$$

2. **market clearing** prize $\lambda^\star$ from

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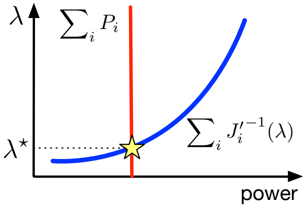

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Auction (dual ascent)

1. local **best response** for given prize:

$$u_i^+ = \operatorname*{argmin}_{u_i \in \mathcal{U}_i} \{J_i(u_i) - \lambda u_i\}$$

2. update prize of **constraint violation**:

$$\lambda^+ = \lambda - \alpha \left(\sum_i P_i + u_i^+\right)$$

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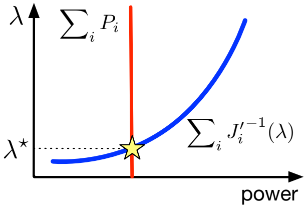

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local (!) $\Longrightarrow$

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measurable (!) $\Longrightarrow$

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$$\lambda^+ = \lambda - \alpha \left(\sum_i P_i + u_i^+\right) = \lambda - \tilde{\alpha} \cdot \omega_{\text{sync}}$$

Continuous-time gather-and-broadcast control

1. $\lambda =$ **aggregate integral** of averaged measurements

$$k\,\dot{\lambda} = -\sum_i C_i\,\omega_i$$

where C_i's are convex and $k > 0$

2. $u_i =$ **local best response** generation dispatch

$$u_i = J_i'^{-1}(\lambda)$$

For quadratic costs: gather-and-broadcast includes ...

1. Automatic Generation Control (AGC): $C_i = \begin{cases} 1 & \text{if } i = i^* \\ 0 & \text{otherwise} \end{cases}$

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2. centralized averaging-based PI (CAPI): $C_i = D_i$

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3. mean-field control: $C_i = 1/n$

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Certificates for gather-and-broadcast applied to DAE model

Theorem I (no assumptions)

- steady-state closed-loop **injections are optimal**

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Scaled cost functions

strictly convex, cont. diff. function J with

$J'(0) = 0$; $\lim_{u \to \partial\mathcal{U}} J(u) = \infty$; $J_i(\cdot) = J\left(\frac{1}{C_i}\cdot\right)\ \forall i$

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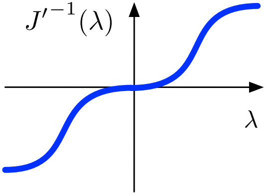

$\Rightarrow$ scaled response curve
$J_i'^{-1}(\lambda) = C_i \cdot J'^{-1}(\lambda)$

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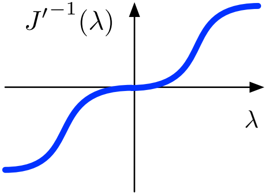

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Theorem II (for scaled cost functions)

- **asymptotic stability** of closed-loop equilibria $|\theta_i^* - \theta_j^*| < \pi/2 \ \forall \{i, j\}$
- frequency regulation & optimal economic dispatch **problems solved**

Hamilton, Bregman, Lyapunov, Luré, & LaSalle invoked

1. incremental, dissipative Hamiltonian, & DAE system

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega}$$

$$\boldsymbol{M}\dot{\boldsymbol{\omega}} = -\boldsymbol{D}\boldsymbol{\omega} - \left(\nabla U(\boldsymbol{\theta}) - \nabla U(\boldsymbol{\theta}^*)\right) + \left(\boldsymbol{J}'^{-1}(\lambda) - \boldsymbol{J}'^{-1}(\lambda^*)\right)$$

$$k\,\dot{\lambda} = -\boldsymbol{c}^\top \boldsymbol{\omega}$$

2. Lyapunov function: energy function + Bregman divergence

$$\mathcal{H}(\boldsymbol{\theta}, \boldsymbol{\omega}, \lambda) := U(\boldsymbol{\theta}) - U(\boldsymbol{\theta}^*) - \nabla U(\boldsymbol{\theta}^*)\,(\boldsymbol{\theta} - \boldsymbol{\theta}^*) + \frac{1}{2}\boldsymbol{\omega}^\top \boldsymbol{M}\boldsymbol{\omega} + \mathcal{I}(\lambda) - \mathcal{I}(\lambda^*) - \mathcal{I}'(\lambda^*)\,(\lambda - \lambda^*)$$

3. Luré integral

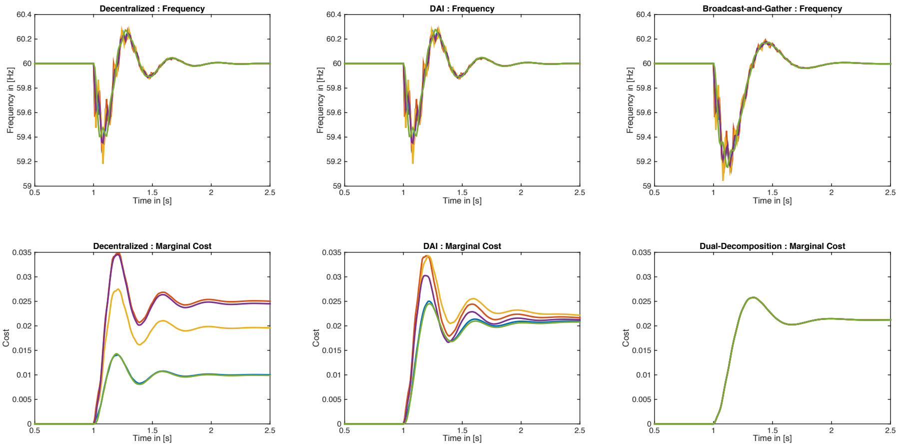
$$\mathcal{I}(\lambda) := k\int_{\lambda_0}^{\lambda} J'^{-1}(\xi)\,\mathrm{d}\xi$$

4. LaSalle invariance principle for DAE systems

J. Schiffer and F. Dörfler. "On stability of a distributed averaging PI frequency and active power controlled differential-algebraic power system model". *European Control Conference*, 2016.

case study: IEEE 39 New England system

Comparison of different frequency control strategies



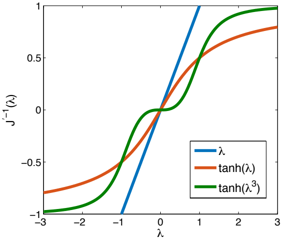
(idealized) decentralized integral control

distributed averaging integral (DAI) control

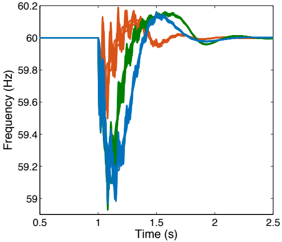
gather-and-broadcast integral control

gather-and-broadcast is comparable to DAI with much less communication

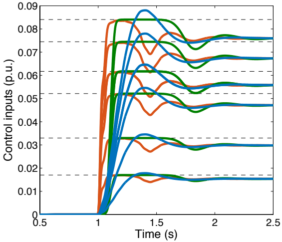
Effect of nonlinear frequency response curves



optimal local frequency response curves $J'^{-1}(\lambda)$



gather-and-broadcast closed-loop frequencies



gather-and-broadcast control inputs

Conclusions

Summary:

- nonlinear, differential-algebraic, heterogeneous power system model
- critical review of decentralized → distributed → centralized architectures
- competitive market ⇒ inspires dual ascent ⇒ gather-and-broadcast
- scaled cost functions ⇒ asymptotic stability & optimality of closed loop

Open problem:

- remove assumption on scaled cost functions

Future work:

- incorporate forecasts & inter-temporal constraints

Dörfler, Grammatico, *Gather-and-broadcast frequency control in power systems*, Automatica, 2017.