Electricity price forecasting: from probabilistic to deep learning approaches

TU Delft & VITO-Energyville

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Outline

Outline

Who Am I?

Personal Information

- Researcher at Energyville-VITO.
- Last-year PhD student at TU Delft.
- **Research topic:** algorithms for electricity markets that help increase integration of renewable energy sources (RES).

Outline

Research Topic

Problem

- Generation of RES is uncertain due to weather dependence.
- As RES penetration increases:
  1. Electricity prices becomes more volatile.
  2. Imbalances between generation and consumption increase.

Solution

Control algorithms for energy systems and electricity markets that:

1. Reduce negative effects of RES integration.
2. Increase the profitability of RES.

Role of Forecasting

Importance of Forecasting

- Forecasting is key to develop these control algorithms.
- Knowledge of future prices allows (among others):
    1. Control RES systems to maximize profits.
    2. Reduce risks by hedging against uncertainties.
    3. Solve stochastic economic dispatch problems.

Outline

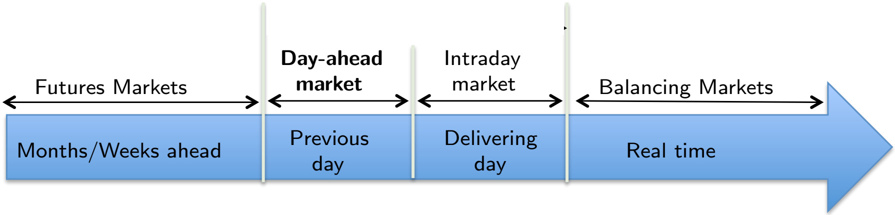
Focus of the talk

Electricity Markets

- Electricity is traded in several sequential markets.

Topic of the Talk

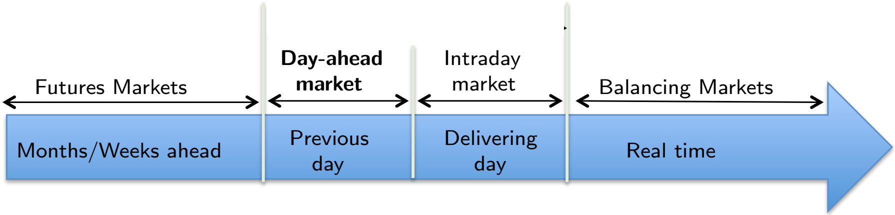
- Day-ahead price forecasting



Focus of the talk

Motivation

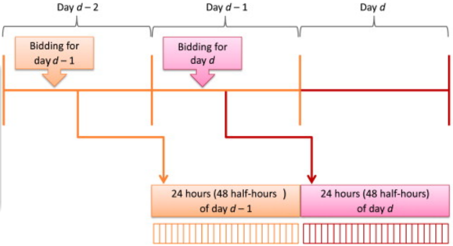
- More volatile than futures and more liquid than intraday
- Large amount of RES traded on it
- Most of the literature focus on the day-ahead market
- Described methods apply to other markets



Day-ahead forecasting

Definition

- Before deadline in day $d-1$, predict the 24 (48) day-ahead prices of day d.



Source: Electricity price forecasting: A review of the state-of-the-art with a look into the future

Literature

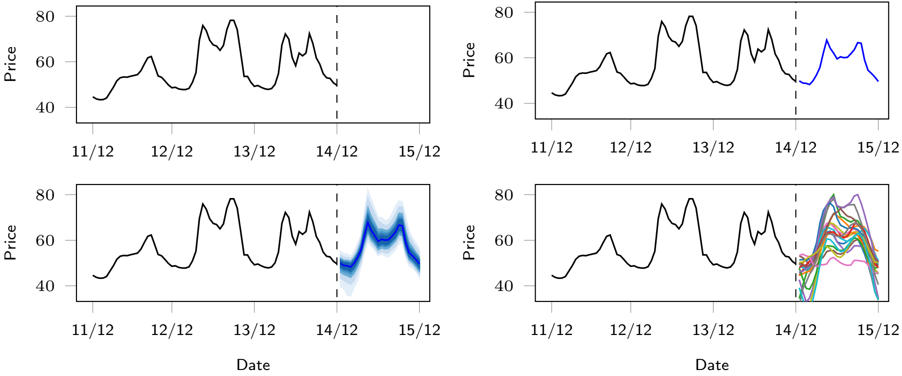
- 20-30 years old field with numerous and diverse methods:
  - Multi-agent models
  - Fundamental models
  - Statistical & machine learning models → Most accurate
- This talk: we focus on statistical & machine learning models

Outline

Types of forecasting

Time series forecasting

- The forecast type depends on the type of information needed:
  - **Point forecast:** expected prices
  - **Probability forecast:** price distribution
  - **Scenario forecast:** possible price realizations



Outline

Point forecasting

Definition

- Point forecast only represent expected price
- It does not model uncertainty, e.g. forecasting error
- It cannot be used for assessing risks

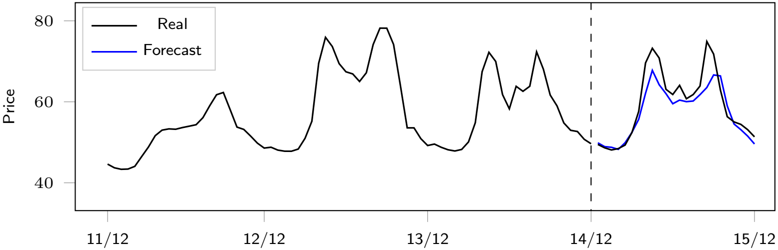


Fig: Day-ahead point forecast for the 14/12/2018 in the Nordpool

Outline

Probability forecasting

Definition

- Probability forecast represent price distribution
- It models the uncertainty of the forecasting error
- Two disadvantages:
  1. Hard to use in stochastic optimization problems
  2. No correlation between prices → unrealistic samples

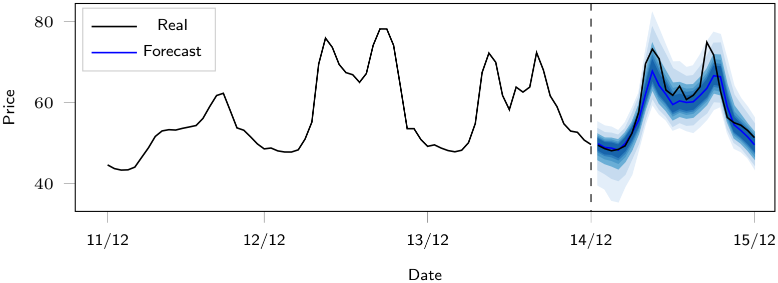


Fig: Day-ahead probability forecast for the 14/12/2018 in the Nordpool

Outline

Scenario Generation forecasting

Definition

- Scenarios represent possible price realizations
- They model not just uncertainty but also correlation
- Easy to use in stochastic optimization problems

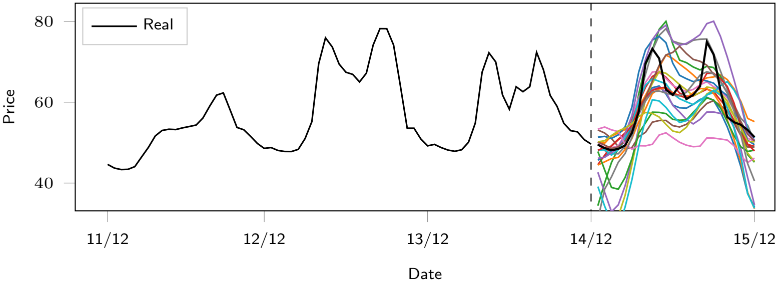


Fig: Day-ahead price scenarios for the 14/12/2018 in the Nordpool

Outline

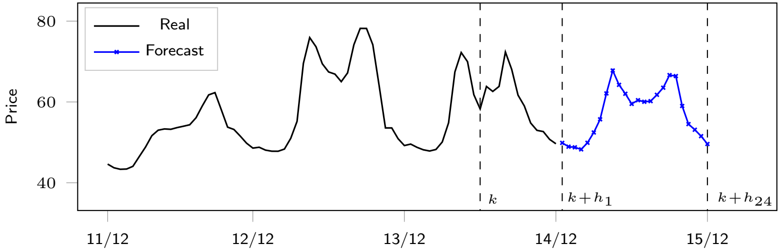
Definition

Day-ahead point forecast

- Expected price p at time $k+h$ estimated at time k:

$$\hat{p}_{k+h} = M(\theta, \mathbf{x}_k)$$

- $\hat{p}$: expected value of p
- θ: model parameters
- k: midday previous day
- $\mathbf{x}$: model inputs
- M: forecast model
- 24 horizons $h_1, \ldots, h_{24}$

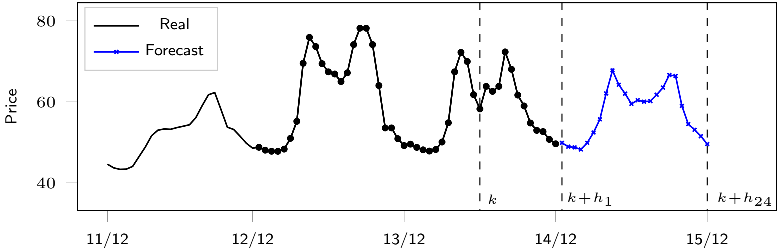


Model inputs

Definition

Inputs $\mathbf{x}_k$ defined by two types of data:

1. Historical prices at previous days, i.e. $p_{d-1}, \ldots, p_{d-n_d}$
2. Exogenous inputs:
   - Wind power forecast day d
   - Load forecast for day d



Type of Models

Types of models

Literature very large: numerous and different methods.

Families of methods

Techniques are usually divided into two families:

1. **Statistical methods:** ARIMA, ARMAX, ARX...
2. **Machine learning methods:** neural nets, regression trees...

Combining models

Combining different types of models improves accuracy (not covered here)[a]

[a] Nowotarski, Raviv, et al., "An empirical comparison of alternative schemes for combining electricity spot price forecasts".

Outline

Statistical Methods - Definition

Properties

- No clear definition of a statistical method
- Sometimes the same as some machine learning methods
- In price forecasting, they are defined by their properties:
  1. Linear models.
  2. Usually including autoregressive terms.
  3. Sometimes including moving average terms.
  4. Designed to include seasonal patterns.

Example

ARX

- Linear model that considers:
  1. **Autoregressive inputs**
  2. **Seasonal components**
  3. **Exogenous inputs**
- Example:

$$\hat{p}_{d,h} = \theta_1 \cdot p_{d-1,24} + \ldots + \theta_{m_1} \cdot p_{d-7,h} + \theta_{m_1+1} \cdot z_1 + \ldots + \theta_{m_1+n} \cdot z_n$$

Statistical Methods - Pros and Cons

- Advantages
  - ✓ Easy and fast to implement and estimate
  - ✓ For pure time series data, i.e. no exogenous inputs, they typically outperform machine learning methods
  - ✓ Small parameter number $\implies$ for small datasets they outperform machine learning methods
- Drawbacks
  - ✗ Sometimes too simple for the nonlinear dynamics of prices. Not good for markets with rapid variations and high frequency changes
  - ✗ If prices depend on several exogenous inputs; e.g. demand, or generation; they might not model the complex relations.

Statistical Methods - State of the art

- State-of-the-art statistical method: fARX-Lasso[a]
- Evaluated in multiple markets[abc]
  1. Always better than other statistical methods
  2. Sometimes better than machine learning methods
  3. Sometimes worse than machine learning methods
- So many ARX in literature, how is this different?
  - Literature models had limited input features
  - 200+ input features + implicit feature selection via LASSO

[a] Uniejewski, Nowotarski, et al., "Automated variable selection and shrinkage for day-ahead electricity price forecasting"

[b] Uniejewski and Weron, "Efficient Forecasting of Electricity Spot Prices with Expert and LASSO Models"

[c] Lago et al., "Forecasting spot electricity prices: deep learning approaches and empirical comparison of traditional algorithms"

Statistical Methods - State of the art

fARX-Lasso Improvements

- Variance stabilization transformation[a]
- Average over different calibration windows[b]

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[a] Uniejewski and Weron, "Efficient Forecasting of Electricity Spot Prices with Expert and LASSO Models".

[b] Marcjasz et al., "Selection of Calibration Windows for Day-Ahead Electricity Price Forecasting".

Outline

Machine Learning (ML) - Outline

Outline

While the field of ML is extensive, forecasting of electricity prices is usually based in one of three family of methods:

1. Neural networks
2. Ensemble of trees
3. Support vector regressors

Machine Learning (ML) vs Statistical Methods (SM)

Summary

- Not clear whether ML methods are better than SM
  - Some studies have shown ML being better
  - Many others have shown SM being better
- In general, the best model depends on
  1. Dataset/market under study
  2. Period under study
  3. Type and number of exogenous inputs
- Several studies have shown neural nets perform poorly

Machine Learning - Pros and Cons

- Advantages
  - ✓ Better suitable for prices with complex nonlinear dynamics, e.g. prices with rapid variations or prices with frequent and large spikes
  - ✓ They can better model the complex relation between some exogenous inputs and prices, e.g. prices in neighboring markets
  - ✓ Estimation times larger than most statistical methods, but with current standard laptop hardware, below 10 minutes.

- Drawbacks
  - ✗ For pure time series data, i.e. no exogenous inputs, they are overkilling and underperform statistical methods
  - ✗ Large number of parameters $\implies$ they require larger datasets than statistical methods.
  - ✗ Harder to interpret input-output relations.

Outline

Deep Learning (DL) - Outline

Motivation

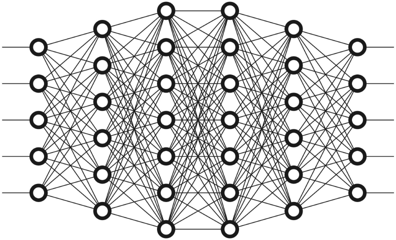
- In recent years, several studies have shown deep learning (DL) models being better than traditional ML and statistical methods
- Natural question 1: what is DL?
- Natural question 2: are DL methods really better?

What is deep learning?

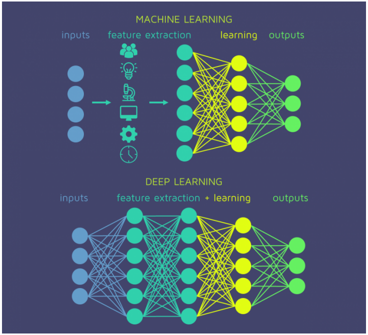
Deep Learning vs Machine Learning

Two main and complementary views:

- Large neural networks
- Feature learning, a.k.a. automatic feature extraction



Source: towardsdatascience.com/why-deep-learning-is-needed-over-traditional-machine-learning-1b6a99177063

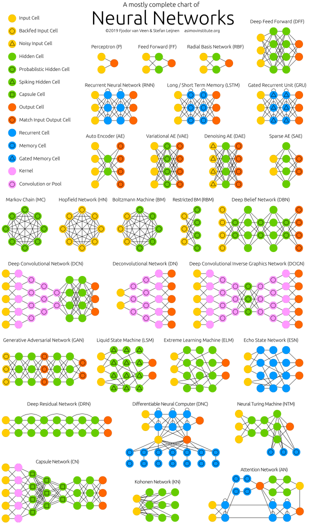


Source: quantdare.com/what-is-the-difference-between-deep-learning-and-machine-learning

What is deep learning?

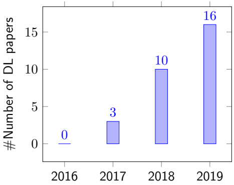
Definition

- **Wikipedia:** artificial neural networks with multiple hidden layers that can extract higher level features
- New classes of neural networks
- New complementary algorithms:
  - New regularization techniques, e.g. dropout
  - New optimizers, e.g. Adam
  - New activation functions, e.g. ReLU



Source: Van Veen, F. & Leijnen, S. (2019). The Neural Network Zoo

DL for electricity price forecasting



Observation I

New DL methods for price forecasting are continuously being proposed

Observation II

Most of them claim to have state-of-the-art result. Yet:

- Most only used 2-4 benchmark models (all based on ML)
- None compared with fARX-Lasso (state-of-the-art statistical method)

Motivation

We proposed new DL methods and performed an extensive comparison[a]:

- 23 literature models (inc. fARX-Lasso) + commercial software

[a] Lago et al., "Forecasting spot electricity prices: deep learning approaches and empirical comparison of traditional algorithms".

DL Case Study - Definition

Study Description

- 4 DL models proposed:
  - Deep feedforward network (DNN)
  - 2 Recurrent network (LSTM and GRU)
  - Convolutional network (CNN)
- Evaluated BELPEX (Belgian) market
  - High forecasting errors and volatile prices
  - Difficult market for statistical methods
- Comparison against 23 literature models + commercial software

Study importance

- Remains to date as the only comparison of DL against several statistical methods
- Remains to date as the only comparison of DL against fARX-Lasso

DL Case Study - Results

3/4 DL models better than literature

Performance separation between ML and SM

fARX-Lasso as good as traditional ML

Model	SMAPE [%]	Class
DNN	12.34	ML
GRU	13.04	ML
LSTM	13.06	ML
MLP	13.27	ML
SVR	13.29	ML
SVR-SOM	13.36	ML
SVR-ARIMA	13.39	ML
GBT	13.74	ML
fARX-EN	13.76	SM
CNN	13.91	ML
fARX-Lasso	13.92	SM
Commercial	14.11	
RBF	14.77	ML
fARX	14.79	SM
RF	15.39	ML
IHMARX	16.72	SM
DR	16.99	SM
TARX	17.08	SM
ARX	17.34	SM
SNARX	17.58	SM
TBATS	17.9	SM
ARIMA-GARCH	19.3	SM
AR	19.31	SM
DSHW	19.4	SM
WARIMA-RBF	22.82	SM
WARIMA	22.84	SM

DL Case Study - Discussion

DL Performance

- Why the DL models performed so good?
  1. Market under study
  2. Depth and number of neurons (previous studies used shallow networks)
  3. Dataset size
  4. Optimization method
- Paper[a] provides analysis of these factors (not discussed here)

[a] Lago et al., "Forecasting spot electricity prices: deep learning approaches and empirical comparison of traditional algorithms".

Statistical Methods vs Machine Learning

Statistical methods performed worse than most ML. However:

- BELPEX is a difficult market for statistical methods
- fARX-Lasso still performs as good as traditional ML

DL - Summary

1. DL models might obtain state-of-the-art results
2. However, this might be conditioned to different factors, e.g. market under study or exogenous inputs
3. Experimental results limited: more studies needed
   - Current work: evaluation of deep neural networks against fARX-Lasso for Nordpool, PJM, and EPEX-FR.

Outline

Point Forecasting - Summary

1. No method is the best under all conditions
2. Best model will depend on different factors, e.g.
   - Without exogenous inputs, DL or ML are overkilling
   - For complex price dynamics, statistical methods might not suffice
3. DL models might obtain state-of-the-art results conditioned to these factors

Outline

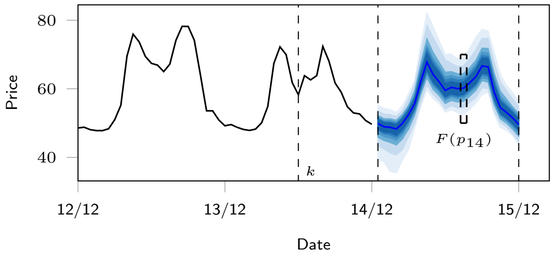
Definition

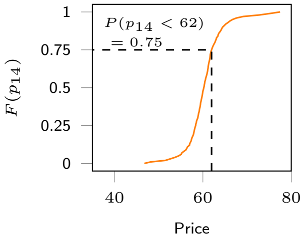
Day-ahead probability forecast

- Cumulative distribution (CDF) of price p at time h estimated at k:

$$F(p_h) = M(\theta, \mathbf{x}_k)$$

- $F(p)$: estimated CDF of p
- θ: model parameters
- k: midday previous day
- $\mathbf{x}$: model inputs
- M: probabilistic model
- 24 CDFs: $F(p_1), \ldots, F(p_24)$





Outline

Probability forecasting methods

Types of models

Two families of methods:

- Parametric models
- Quantile models

Parametric models

- The forecast is given by a full parameterization of the probability distribution, e.g:

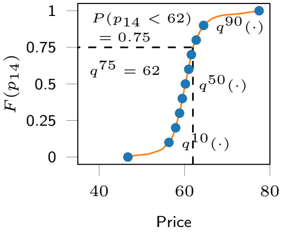
$$p_h \sim \mathcal{N}(\mu_{p_h}, \sigma_{p_h})$$

- Two main parametric distributions
  - Johnson's S_U distribution
  - Skew-t distribution

Probability forecasting methods

Quantiles

- Define random variable p and its CDF $F(p)$.
- Quantile q^α of p is the value at which the probability of p is less than or equal to α, i.e. $\alpha = F(q^\alpha)$.



Quantile functions

- $F(p)$ is approximated building quantiles models $q^\alpha(\theta, \mathbf{x})$
- 4 main methods exist:
  1. Empirical quantiles
  2. Quantile regression
  3. Quantile regression averaging
  4. Bootstrapping

Parametric vs Quantile Methods

1. Parametric models expected to perform worse than quantile models[a]
   - Parametric models limited by distribution assumption

2. Recent study: parametric model performs similar to quantile method[a]
   - Skew-t distribution slightly better than linear quantile regression

   However:
   - Other parametric models were worse than quantile model
   - Quantile model similar performance as the best parametric
   - Best quantile model not considered

3. Here we focus on quantile methods
   - More general: no assumptions needed
   - Often better accuracy

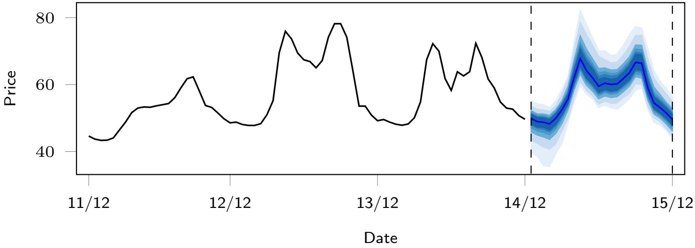
[a] Gianfreda et al., "A stochastic latent moment model for electricity price formation"

Outline

Prediction intervals vs Quantiles

Difference

- Some probabilistic forecasting papers provide prediction intervals (PI)
- Some others provide quantiles q^{α}
- Quantiles are just a generalization of prediction intervals
  - Example: 90% PI equals the interval $[q^5, q^{95}]$
- In this talk, we use the word quantile as a general term



Quantile Method I - Empirical Quantiles

Algorithm

1. Consider past point forecasts at hour h:

$$\left[\hat{p}_{d,h}, \ldots, \hat{p}_{d-n,h}\right]^\top$$

2. Compute historical forecasting errors $\epsilon_d, \ldots, \epsilon_{d-n}$.
3. Compute empirically quantile distribution $q^\alpha(\epsilon)$ of errors.
4. Quantile function of price at hour h given by:

$$q^\alpha(p_{d,h}) = \hat{p}_{d,h} + q^\alpha(\epsilon)$$

   i.e. point prediction plus quantile function of errors.

Quantile Method II - Quantile Regression

Quantile Regression

- Parameterizes quantile functions $q^\alpha(\cdot)$ by model $M^\alpha(\theta, \mathbf{x})$.
- Estimates $M^\alpha(\theta, \mathbf{x})$ by solving:

$$\min_{\theta} \sum_{i=1}^{N} (\alpha - 1) \max\big(0, M^\alpha(\theta, \mathbf{x}_i) - p_i\big) + \alpha \max\big(0, p_i - M^\alpha(\theta, \mathbf{x}_i)\big)$$

  where:

  - $\{(\mathbf{x}_i, p_i)\}_{i=1}^N$ dataset of prices and inputs
  - Inputs $\mathbf{x}_i$ the same as for point forecasts

Examples

- Most common model: linear quantile regression $M^\alpha(\theta, \mathbf{x}) = \theta^\top \mathbf{x}$
- Nonlinear version: $M^\alpha(\theta, \mathbf{x})$ as a neural network

Method III - Quantile Regression Averaging

Quantile Regression Averaging (QRA)

- Estimate quantiles using point forecasts and linear quantile regression:

  1. Build N different point forecasts
  2. Use the N predictions as vector of input features $\mathbf{x}$
  3. Apply standard quantile regression

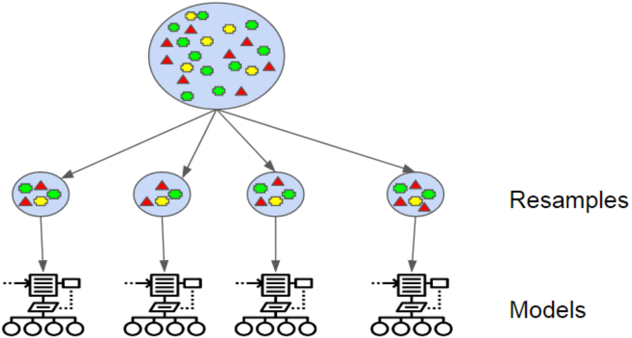
Motivation

Estimate quantiles for nonlinear dynamics with linear method

Method IV- Bootstrapping

Conceptual Idea

1. Generate datasets obtained via resampling with replacement
2. Estimate a point forecast $\hat{p}_h$ model for each dataset



Source: https://hub.packtpub.com/ensemble-methods-optimize-machine-learning-models/

Method IV- Bootstrapping

Conceptual Idea

1. Generate datasets obtained via resampling with replacement
2. Estimate a point forecast $\hat{p}_h$ model for each dataset
3. Use models to estimate quantiles $q^\alpha(\epsilon_\mathrm{m})$ of model errors
4. Use $q^\alpha(\epsilon_\mathrm{m})$ to estimate quantiles $q^\alpha(\epsilon_\mathrm{p})$ of process errors
5. Quantile function of price at hour h given by:

$$q^\alpha(p_h) = \mathbb{E}\{\hat{p}_h\} + q^\alpha(\epsilon_\mathrm{p}) + q^\alpha(\epsilon_\mathrm{m})$$

Characteristics

- It distinguishes between model and process errors
- More computationally demanding than the others

Quantile Methods - State of the art

- Several studies have evaluated and compare quantile methods, e.g.
  1. Global Energy Forecasting Competition (GEFCom2014)
  2. Different papers[a][b][c]

- State-of-the-art method: Quantile regression averaging (QRA)

- Inexistent bootstrapping vs QRA comparison.
  - In our experience, bootstrapping performs worse.

[a] Nowotarski and Weron, "Recent advances in electricity price forecasting: A review of probabilistic forecasting"

[b] Uniejewski, Marcjasz, et al., "On the importance of the long-term seasonal component in day-ahead electricity price forecasting: Part II — Probabilistic forecasting"

[c] Maciejowska et al., "Probabilistic forecasting of electricity spot prices using Factor Quantile Regression Averaging"

Outline

New Interesting Trend I - Methods better than QRA

Recent Developments

Last months: 2 approaches that performs similar to QRA were proposed:

1. A Bayesian stochastic volatility model[a]
2. A conformal prediction model[b]

Importance

Methods that perform in some cases better than the state-of-the-art

Word of caution

- QRA still perform similar to these two
- New methods tested in one study, QRA in many of them

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[a] Kostrzewski et al., "Probabilistic electricity price forecasting with Bayesian stochastic volatility models"

[b] Kath et al., "Conformal Prediction Interval Estimations with an Application to Day-Ahead and Intraday Power Markets"

New Interesting Trend II - Deep Learning (DL)

Summary

- Research on DL for probability forecasting is very limited
- Many claim to do DL for probability forecasting[a], however
  - DL is only used for point forecasting
  - Prob. forecasting is made based on standard methods

[a] Afrasiabi et al., "Probabilistic deep neural network price forecasting based on residential load and wind speed predictions".

Probability Forecasting based on DL

To the best of my knowledge, only two works[a b] in DL

- Drawback: not compared with QRA or other standard methods

[a] Brusaferri et al., "Bayesian deep learning based method for probabilistic forecast of day-ahead electricity prices".

[b] Hu et al., "Distribution-Free Probability Density Forecast Through Deep Neural Networks".

Outline

Probability Forecasting - Summary

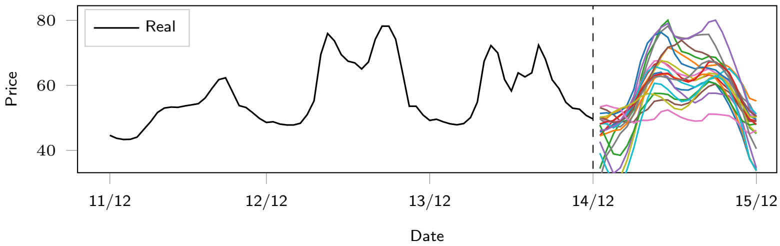
1. Two main family of methods: parametric and quantile models
    - Parametric worse accuracy due to distribution assumption
2. There are several quantile models:
    - State-of-the-art: Quantile regression averaging (QRA)
3. Research on deep learning for probability forecasting is limited
    - Several works use deep learning as a buzzword

Outline

Definition

Scenario generation forecast

- **Recall**: we need possible price realization:
  - Probability functions hard to use in stochastic optimization
  - Probability functions do not consider price correlations
- **Goal**: generate N possible price scenarios $S^1, \ldots, S^N$
  - $S^i = [p^i_1, \ldots, p^i_{24}]$: possible realization of day-ahead prices
  - Scenarios with marginal distributions equal to probability forecasts



Scenario Generation - Literature

Observations

1. Specific scenario generation literature is scarce
2. In general, existing papers do not propose new methods:
   - Their research goal is to solve a stochastic optimization problem, e.g. optimal market bidding
   - They consider a generic scenario generation method
3. No paper comparing different methods

Consequences

- Many methods could be presented
- It would be hard to draw comparisons
- We briefly present the main families and explain their differences

Outline

Scenario generation methods

Types of methods

Scenario generation methods used for electricity prices and stochastic optimization can be classified into three families:

1. Sampling-based methods
2. Optimization-based methods
3. Copulas-based methods

Scenario generation - Sampling based methods

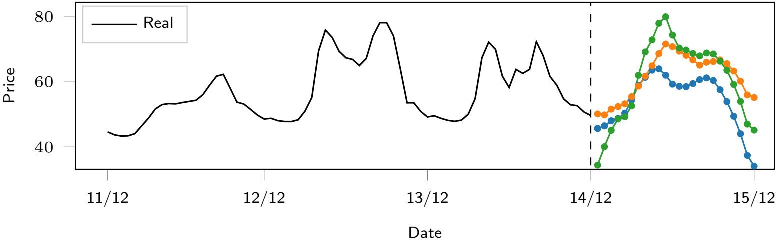
Steps

1. Fit stochastic model to prices
   - ARMA model with Gaussian errors[a]
   - ARMA for prices with GARCH model for conditional error variance[b]
   - Neural network model with Gaussian errors[c]
2. Use stochastic model to recursively simulate scenarios

[a] Fleten et al., "Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer".

[b] Faria et al., "Day-ahead market bidding for a Nordic hydropower producer: taking the Elbas market into account".

[c] Vagropoulos et al., "ANN-based scenario generation methodology for stochastic variables of electric power systems".



Scenario generation - Optimization based methods

Steps

1. Define the statistical metrics of scenarios
   - Metrics usually based on historical data
   - Example: the first four moments of the prices[a][b]
2. Solve optimization problem to generate scenarios:
   - Scenarios as optimization variables
   - Objective: difference between desired and scenario metrics
3. Result: scenarios that satisfy statistical metrics

[a] Høyland et al., "A heuristic for moment-matching scenario generation".

[b] Jensen et al., "A comparison of scenario generation methods for the participation of electric vehicles in electricity markets".

Scenario generation - Copula based methods

Sklar's Theorem applied to prices[a]

- Define the multivariate CDF of the 24 prices by $H(p_1,\ldots,p_{24})$
- Define the marginal CDFs of each price by $F_1(p_1),\ldots,F_{24}(p_{24})$
- There exists a copula function C such that[a]:

$$C\Big(F_1(p_1),\ldots,F_{24}(p_{24})\Big) = H(p_1,\ldots,p_{24})$$

i.e. the marginal and multivariate distributions are related by C

[a] Sklar, "Fonctions de Répartition à n Dimensions et Leurs Marges".

Consequence

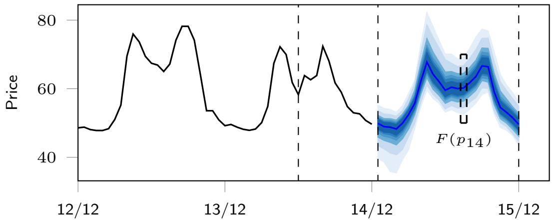
- We can obtain a multivariate distribution based on probability forecasts and sample from it to generate scenarios

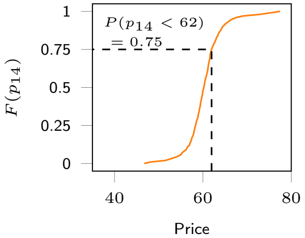
Scenario generation - Copula based methods

Steps

1. Use probability forecasting methods to obtain the marginal distributions $F_1(p_1), \ldots, F_{24}(p_{24})$
2. Define the copula type, e.g.
   - Empirical copula[a]
3. Generate or estimate copula using the marginal distributions
4. Generate scenarios by sampling from the copula

[a] Toubeau et al., "Deep Learning-Based Multivariate Probabilistic Forecasting for Short-Term Scheduling in Power Markets".





Outline

Comparison

- Hard to compare the different methods in terms of accuracy
  - No empirical comparison exists (to the best of my knowledge)
  - We can list the advantages and drawbacks
- Sampling-based methods
  - ✓ Simpler and easier to estimate
  - ✗ Bad approximations with few scenarios
- Optimization based method
  - ✓ Very flexible: generated scenarios can display any desired metrics
  - ✗ Large computational complexity
- Copula-based methods
  - ✓ Generated scenarios follow marginal distributions
  - ✓ For large number of scenarios, less complex than optimization
  - ✗ Distribution depends on selected copula

Outline

Conclusion

1. We have presented forecasting methods for electricity prices
   - Point forecasts
   - Probability forecast
   - Scenario generation methods
2. For the three fields, the best forecasting model depend upon:
   - Market under study
   - Type and size of input data
   - Others
3. Deep learning models are continuously being proposed:
   - Nearly all of them are limited to point forecasting
   - Further comparison against state-of-the-art methods is still needed

Thank you! Any Questions?

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